These students are setting up a tent. How do the students know how to set up the tent? How is the shape of the tent created? How could students find the amount of material needed to make the tent? Why might students want to know the volume of the tent?

What You'll Learn

- Recognize and sketch objects.
- Use nets to build objects.
- Develop and use a formula for the surface area of a triangular prism.
- Develop and use a formula for the volume of a triangular prism.
- Solve problems involving prisms.

Why It's Important

- We find out about our environment by looking at objects from different views. When we combine these views, we have a better understanding of these objects.
- We need measurement and calculation skills to design and build objects, such as homes and parks.
Key Words

- isometric diagram
- pictorial diagram
- triangular prism
- surface area
- volume
Skills You'll Need

Drawing Isometric and Pictorial Diagrams

An isometric diagram shows three dimensions of an object. It is drawn on isometric (triangular) dot paper. Vertical edges of an object are drawn as vertical line segments. Parallel edges of an object are drawn as parallel line segments.

Example 1
Make an isometric diagram of this object.

Solution
On isometric paper, join a pair of dots for each vertical edge. Join a pair of dots diagonally for each horizontal edge that goes up to the right. Join a pair of dots diagonally for each horizontal edge that goes up to the left. Shade the faces so the object appears three-dimensional.

In a pictorial diagram, the depth of an object is drawn to a smaller scale than the length and width. This gives the object a three-dimensional appearance.

Example 2
Make a pictorial diagram of this cylinder.
Solution
The top and bottom of a cylinder are circles.
In a pictorial diagram, a circular face is an oval.
One-half of the bottom circular face is drawn as a broken curve. This indicates that this half cannot be seen.
Draw vertical line segments to join the top and bottom ovals.

✅ Check

1. Make an isometric diagram of each object.
   a) a rectangular prism with dimensions
      3 units by 4 units by 5 units
   b) a square pyramid
   Remember to look up any terms that you are unsure of in the Glossary.

2. Make a pictorial diagram of each object.
   a) a rectangular prism with dimensions 3 units by 4 units by 5 units
   b) a regular tetrahedron

Calculating the Surface Area and Volume of a Rectangular Prism

The surface area of a rectangular prism is the sum of the areas of all its faces.
Since opposite faces are congruent, this formula can be used to find the surface area:
Surface area = $2 \times \text{area of base} + 2 \times \text{area of side face} + 2 \times \text{area of front face}$
Using symbols, we write: $SA = 2lw + 2hw + 2lh$
Since the congruent faces occur in pairs, this formula can be written as:
$SA = 2(lw + hw + lh)$
In this formula, $l$ represents length, $w$ represents width, and $h$ represents height.

The volume of a rectangular prism is the space occupied by the prism.
One formula for the volume is: Volume = area of base $\times$ height
Using symbols, we write: $V = lwh$
Example 3
A rectangular prism has dimensions 4 m by 6 m by 3 m.

a) Calculate the surface area.

b) Calculate the volume.

Solution
Draw and label a pictorial diagram.

a) Use the formula for the surface area of a rectangular prism:

\[ SA = 2(hw + hw + lh) \]

Substitute: \( l = 6 \), \( w = 4 \), and \( h = 3 \)

\[ SA = 2(6 \times 4 + 3 \times 4 + 6 \times 3) \] In the brackets, multiply then add.

\[ = 2(24 + 12 + 18) \]

\[ = 2(54) \]

\[ = 108 \text{ m}^2 \]

The surface area is 108 m².

b) Use the formula for the volume of a rectangular prism:

\[ V = lwh \]

Substitute: \( l = 6 \), \( w = 4 \), and \( h = 3 \)

\[ V = 6 \times 4 \times 3 \]

\[ = 72 \]

The volume is 72 m³.

A cube is a regular polyhedron with 6 square faces.
Since all the faces of a cube are congruent, we can simplify the formulas for surface area and volume. Each edge length is \( s \).
The area of each square face is: \( s \times s = s^2 \)
So, the surface area of a cube is: \( SA = 6s^2 \)
The volume of a cube is: \( V = s \times s \times s = s^3 \)

Check

3. Find the surface area and volume of each rectangular prism. Include a labelled pictorial diagram for each rectangular prism.

a) 12 cm by 6 cm by 8 cm  
b) 7 mm by 7 mm by 4 mm  
c) 2.50 m by 3.25 m by 3.25 m  
d) 5 cm by 5 cm by 5 cm
Calculating the Area of a Triangle

The area of a triangle is calculated with either of these formulas:

\[
\text{Area} = \text{base} \times \text{height} \div 2, \text{ or}
\]

\[
\text{Area} = \text{one-half} \times \text{base} \times \text{height}
\]

Using symbols, we write: \( A = \frac{bh}{2} \) or \( A = \frac{1}{2} bh \)

where \( b \) is the length of the base and \( h \) is the corresponding height.

Example 4

The side lengths of \( \triangle PQR \) are 12 cm, 5 cm, and 13 cm.

\[
\begin{align*}
\text{P} & \quad 12 \text{ cm} \\
\text{R} & \quad 13 \text{ cm} \\
\text{Q} & \quad 4.6 \text{ cm}
\end{align*}
\]

\[ \text{height} \]

\[ \text{base} \]

\[ \text{height} \]

\[ \text{base} \]

a) The height from \( P \) to \( QR \) is about 4.6 cm.

Use this to calculate the area of \( \triangle PQR \).

b) Triangle \( PQR \) is a right triangle with \( \angle P = 90^\circ \).

Use this to calculate the area of \( \triangle PQR \) a different way.

Solution

a) Use the formula: \( A = \frac{bh}{2} \)

Substitute: \( b = 13 \) and \( h = 4.6 \)

\[
A = \frac{13 \times 4.6}{2} = \frac{59.8}{2} = 29.9
\]

The area is 30 cm\(^2\) to the nearest square centimetre.
b) Since \( \triangle PQR \) is a right triangle, the two sides that form the right angle are the base and height. 
\( \angle QPR = 90^\circ \), so \( QP = 5 \text{ cm} \) is the height; and \( PR = 12 \text{ cm} \) is the base. 
Use the formula: \( A = \frac{bh}{2} \) 
Substitute: \( b = 12 \) and \( h = 5 \) 
\[
A = \frac{12 \times 5}{2} = \frac{60}{2} = 30
\]
The area is 30 cm\(^2\).

**Check**

4. Calculate the area of each triangle.
   a) 
   
   b) 
   
   c) 

**Converting among Units of Measure**

\( 1 \text{ m} = 100 \text{ cm} \)

The area of a square with side length 1 m is:
\[
A = 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2
\]

The area of a square with side length 100 cm is:
\[
A = 100 \text{ cm} \times 100 \text{ cm} = 10000 \text{ cm}^2
\]
So, \( 1 \text{ m}^2 = 10000 \text{ cm}^2 \), or \( 10^4 \text{ cm}^2 \)
The volume of a cube with edge length 1 m is:
\[ V = 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} \]
\[ = 1 \text{ m}^3 \]

The volume of a cube with edge length 100 cm is:
\[ V = 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} \]
\[ = 1000000 \text{ cm}^3 \]
So, \( 1 \text{ m}^3 = 1000000 \text{ cm}^3 \), or \( 10^6 \text{ cm}^3 \)

The volume of a cube with edge length 10 cm is:
\[ V = 10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm} \]
\[ = 1000 \text{ cm}^3 \]
Since \( 1 \text{ cm}^3 = 1 \text{ mL} \),
then \( 1000 \text{ cm}^3 = 1000 \text{ mL} \)
\[ = 1 \text{ L} \]

Example 5
Convert.

a) \( 0.72 \text{ m}^2 \) to square centimetres

b) \( 1.05 \text{ m}^3 \) to cubic centimetres

Solution

a) \( 0.72 \text{ m}^2 \) to square centimetres
\[ 1 \text{ m}^2 = 10000 \text{ cm}^2 \]
So, \( 0.72 \text{ m}^2 = 0.72 \times 10000 \text{ cm}^2 \)
\[ = 7200 \text{ cm}^2 \]

To multiply by 10000, move the decimal point 4 places to the right.

b) \( 1.05 \text{ m}^3 \) to cubic centimetres
\[ 1 \text{ m}^3 = 1000000 \text{ cm}^3 \], or \( 10^6 \text{ cm}^3 \)
So, \( 1.05 \text{ m}^3 = 1.05 \times 10^6 \text{ cm}^3 \)
\[ = 1050000 \text{ cm}^3 \]

This answer is in scientific notation.

This answer is in standard form.

Check

5. Convert. Write your answers in standard form and in scientific notation, where appropriate.

a) \( 726.5 \text{ cm} \) to metres
b) \( 4300 \text{ cm}^2 \) to square metres
c) \( 980000 \text{ cm}^3 \) to cubic metres
d) \( 4280000 \text{ cm}^3 \) to litres
e) \( 8.75 \text{ m} \) to centimetres
f) \( 1.36 \text{ m}^2 \) to square centimetres
g) \( 14.98 \text{ m}^3 \) to cubic centimetres
h) \( 9.87 \text{ L} \) to cubic centimetres
When we draw a view of an object, we show internal line segments only where the depth or thickness of the object changes. Here is an object made with 7 linking cubes.

Here are the views:

The broken lines show how the views are aligned.

Explore

Work in a group of 3.
Each student needs 8 linking cubes and isometric dot paper.
Each student chooses one of these views.

You use 8 cubes to build an object that matches the view you chose. Sketch your object on isometric paper. Use your cubes to build, then sketch, a different object with the view you chose.

Reflect & Share

Compare your objects with those of other members of your group. Does any object match all 3 views? If not, build and sketch one that does. What helped you decide the shape of the object? Are any other views needed to identify the object? Explain.
Each view of an object provides information about the shape of the object. When an object is built with linking cubes, the top, front, and side views are often enough to identify and build the object. These views are drawn with the top view above the front view, and the side views beside the front view, as they were at the top of page 102. In this way, matching edges are adjacent.

Example

Which object has these views?

Solution

The top view is a regular hexagon. The side view is 2 congruent rectangles. The front view is 3 rectangles, 2 of which are congruent. This object is a hexagonal prism.
You will need linking cubes, isometric dot paper, and grid paper.

1. Match each view A to D with each object H to L.
   Name each view: top, bottom, front, back, left side, or right side

2. Sketch a different view of two of the objects in question 1.

3. Use these 4 clues and linking cubes to build an object.
   Draw the object on isometric paper.
   Clue 1: There are 6 cubes in all. One cube is yellow.
   Clue 2: The green cube shares one face with each of the other 5 cubes.
   Clue 3: The 2 red cubes do not touch each other.
   Clue 4: The 2 blue cubes do not touch each other.

4. a) Build the object for the set of views below.
   b) Sketch the object on isometric dot paper.
5. a) Build an object with each number of linking cubes.
   For 3 or more cubes, do not make a rectangular prism.
   i) 2  ii) 3  iii) 4  iv) 5  v) 6
b) For each object you build, count the number of its faces, edges, and vertices. Record your results in a table.
c) Look for a pattern in the table in part b.
   For any object, how are the numbers of faces, edges, and vertices related?
d) Build an object with 7 linking cubes.
   Check that the relationship in part c is true.

6. **Assessment Focus**
   a) Use these views to build an object.

   ![Top View and Right Side View of an Object]

   b) Sketch the object on isometric dot paper.
   c) Draw the other views of the object.

7. a) Use these views
to build an object.
   A shaded region has no cubes.
b) Sketch the object.
   Explain your work.

   ![Top View and Front View of an Object]

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The relationship in part c is called Euler's formula. It is named for a Swiss mathematician, Leonhard Euler, who lived in the 18th century.

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**Reflect**

How do views help to show a three-dimensional object?
Use an example to explain.
A net is a pattern that can be folded to make an object.

Here is a net and the rectangular prism it forms.

A polyhedron can have several different nets.

**Explore**

Work with a partner.
You will need scissors, tape, and 1-cm grid paper.
For each set of views below:

- Identify the object. Draw a net of the object.
- Cut out your net. Check that it folds to form the object.
- Describe the object.

**Set A**

**Top View**

- 6 cm

**Front View**

- 5 cm

**Right Side View**

- 2.5 cm
- 6.5 cm

**Set B**

**Top View**

- 6 cm

Remember that an internal line segment on a view shows that the depth changes.
Reflect & Share

Compare your nets with those of another pair of classmates.
How did you know how many faces to draw?
How do you know which faces share a common edge?
How do the faces on the net compare to the views for each object?
Could you have drawn a different net for the same object? Explain.

Connect

Some faces are not visible from a particular view.
Look at the views of an object, at the right.

The top view shows that four congruent triangular faces meet at a point.
The front and side view shows an isosceles triangular face.
The bottom view shows the base is a square.
This object is a square pyramid.
We can use a ruler, protractor, and compass to draw the net.

In the centre of the paper, draw a 5-cm square for the base.
On each side of the base, draw an isosceles triangle with two equal sides of 6 cm.

Other arrangements of the five faces may produce a net.
Each outer edge must match another outer edge, and no faces must overlap.
The net at the right is constructed so that the 5-cm square base is attached to only one triangle.
Each net folds to form a square pyramid.

Many views of an object are needed to create its net, especially if the object is made with linking cubes.

**Example**

These four views show an object made with linking cubes. Use the views to draw a net for the object.

**Solution**

Use 1-cm dot paper. Start with the simplest view, which is the bottom view. Draw the bottom face. Draw the front face and back face above and below the bottom face. The top view and side views show a change in depth. A new face is drawn for each depth change. A side square is drawn for each depth change.

Each side view is drawn to touch the square that shows the change in depth.
The net in the *Example* folds to form this object with 10 faces, including the bottom face.

The symmetry of the object made it easier to draw a net. Other arrangements of the 10 faces may be folded to form the object.

**Practice**

1. Each set of views below represents an object.
   a) Identify each object.
   b) Draw a net for the object.
   c) Cut out the net. Build the object.
   d) Describe the object.
   
   ![Diagram](image)

2. Each set of views represents an object. Draw 2 different nets for each object. Build the object.
   a) ![Diagram](image)
   b) ![Diagram](image)

3. Choose one set of views from question 2. Describe the steps you used to draw the net.
4. A chocolate box has the shape of a prism with a base that is a rhombus. Each side length of the rhombus is 3.6 cm. The angles between adjacent sides of the base are 60° and 120°. The prism is 10.8 cm long.
   a) Draw a net for this box.
   b) How is your net different from the cardboard net from which the box is made? Explain.

5. These views represent an object. Each shaded region is an opening in a face. Draw a net for the object. Build the object.

6. **Assessment Focus**
   These views represent an object.
   a) Identify the object.
   b) Draw two different nets.
   c) Use the two different nets to build the object.
   d) Is one net easier to draw or fold? Explain.

**Reflect**
Describe how a set of views of an object relates to the figures on a net of the object. Is there one correct net for a set of views? Explain with an example.
**Mid-Unit Review**

**LESSON**

**3.1** 1. a) Use linking cubes. Use the views below to build the object. Remember that internal lines show where the depth of the object changes.

   ![Top View](image1)
   ![Left Side View](image2)
   ![Front View](image3)
   ![Right Side View](image4)

   b) Sketch the object on isometric dot paper.

2. a) Use the views below to describe the object. Remember that a shaded area shows an opening in the face.

   ![Top View](image5)
   ![Front View](image6)
   ![Right Side View](image7)
   ![Front/Side View](image8)

   b) Sketch a pictorial diagram of the object.

**3.2** 3. Each set of views that follows represents an object.

   a) Identify each object.
   b) Draw a net for each object.
   c) Cut out the net. Build the object.
   d) Describe the object.

   i) ![Top View](image9)

   ii) ![Top View](image10)

   iii) ![Top View](image11)
A triangular prism is formed when a triangle is translated in the air so that each side of the triangle is always parallel to its original position.

The two triangular faces are the bases of the prism.

**Explore**

Work with a partner.
You will need 1-cm grid paper.
➢ For each triangular prism below:
  Draw a net.
  Find the surface area of the prism.

The surface area of an object is the sum of the areas of its faces.

➢ Write a formula you can use to find the surface area of any triangular prism.

**Reflect & Share**

Compare your nets and formula with those of another pair of classmates.
Did you write the same formula?
If not, do both formulas work? Explain.
Did you write a word formula or use variables? Explain.
Here is a triangular prism and its net. Both the prism and net are drawn to scale.

The two triangular faces of the prism are congruent. Each triangular face has base 11 cm and height 3 cm. So, the area of one triangular face is: \( \frac{1}{2} \times 11 \times 3 \)
The surface area of a triangular prism can be expressed using a word formula:

\[
SA = \text{sum of the areas of three rectangular faces} + \text{area of one triangular face}
\]

Use this formula to find the surface area of the prism above.

\[
SA = (5 \times 2) + (11 \times 2) + (8 \times 2) + 2 \times \frac{1}{2} \times 11 \times 3
\]
\[
= 10 + 22 + 16 + 33
\]
\[
= 81
\]

The surface area of the prism is 81 cm\(^2\).

We can use variables to write a formula for the surface area of a triangular prism.

To avoid confusion between the height of a triangle and the height of the prism, we now use \textit{length} instead of \textit{height} to describe the edge that is perpendicular to the base.

For the triangular prism above:
The length of the prism is \( \ell \).
Each triangular face has side lengths \( a, b, \) and \( c \).
The height of a triangular face is \( h \) and its base is \( b \).
The surface area of the prism is:
SA = sum of the areas of the 3 rectangular faces + 2 \times area of one triangular face
SA = al + bl + cl + 2 \times \frac{1}{2} bh
SA = al + bl + cl + bh

**Example**

Find the surface area of the prism below.
Each dimension has been rounded to the nearest whole number.
Write the surface area in square centimetres and in square metres.

**Solution**

Identify the variable that represents each dimension.
Sketch, then label the prism with these variables.
The length of the prism is:
l = 40
The 3 sides of a triangular face are:
a = 20, b = 31, c = 29
The height of a triangular face is:
h = 18

Substitute for each variable in the formula for surface area.
SA = al + bl + cl + bh
= (20 \times 40) + (31 \times 40) + (29 \times 40) + (31 \times 18)
= 800 + 1240 + 1160 + 558
= 3758

The surface area of the prism is 3758 cm².
To convert square centimetres to square metres, divide by 10 000.
3758 cm² = \frac{3758}{10\ 000} \text{ m}^2
= 0.3758 \text{ m}^2

The surface area of the prism is 0.3758 m².
1. Calculate the area of each net.
   a) 
   b) 

2. Calculate the surface area of each prism.
   Draw a net first if it helps.
   Write the surface area in square metres.
   a) 
   b) 

3. Calculate the surface area of each prism.
   The shaded region indicates that the face is missing.
   Write the surface area in square centimetres.
   a) 
   b) 

4. a) What area of wood, in square metres, is needed to make the ramp at the left?
   The ramp does not have a base.
   b) Suppose the ramp is built against a stage.
   The vertical face that is a rectangle is against the stage.
   How much wood is needed now?
5. A plastic container company designs a container with a lid to hold one piece of pie.
   a) Design the container as a triangular prism. Explain your choice of dimensions.
   b) Calculate the area of plastic in your design.

6. The total area of the 3 rectangular faces of an equilateral triangular prism is 72 cm².
   a) No dimension can be 1 cm. What are the possible whole-number dimensions of the edges?
   b) Sketch the prism with the greatest length. Include dimensions. Explain your choice.

7. **Assessment Focus** How much metal, in square metres, is needed to build this water trough?

8. A right triangular prism has a base with perimeter 12 cm and area 6 cm².
   a) Find the whole-number dimensions of the base.
   b) The length of the prism is 6 cm. Calculate the surface area of the prism.
   c) Sketch the prism. Include its dimensions.

**Take It Further**

9. Use the variables below to sketch and label a triangular prism. The lengths have been rounded to the nearest whole number. Calculate the surface area of the prism.
   \( a = 7 \text{ cm}, \ b = 17 \text{ cm}, \ c = 11 \text{ cm}, \ h = 3 \text{ cm}, \ l = 12 \text{ cm} \)

**Reflect**

Write to explain how to find the surface area of a triangular prism. Include an example and a diagram.
Recall that the area of a triangle is one-half the area of a rectangle that has the same base and height. That is, the area of $\triangle DEC = \frac{1}{2}$ the area of rectangle $ABCD$.

**Explore**

Work in a group of 4.
You will need 4 identical cereal boxes, one for each group member, and markers.

- Find the volume of your cereal box, which is a rectangular prism.
- Use a ruler to draw a triangle on one end of the cereal box.
The base of each triangle should be along one edge of the box.
The third vertex of the triangle should be on the opposite edge. Make sure you draw different triangles.
What is the volume of a triangular prism with this triangle as its base, and with length equal to the length of the cereal box?

- Compare your answer with those of other members of your group. What do you notice?
- Work together to write a formula for the volume of a triangular prism.

**Reflect & Share**

How is the volume of a triangular prism related to the volume of a rectangular prism?

Compare your formula for the volume of a triangular prism with that of another group. Did you use variables in your formula? If not, work together to write a formula that uses variables.

**Connect**

The volume of a rectangular prism is:

$$V = \text{base area} \times \text{length}$$
Suppose we draw a triangle on the base of the prism so that the base of the triangle is one edge, and the third vertex of the triangle is on the opposite edge.

The volume of a triangular prism with this base, and with length equal to the length of the rectangular prism, is one-half the volume of the rectangular prism.

Since the base area of the triangular prism is one-half the base area of the rectangular prism, the volume of a triangular prism is also: \( V = \text{base area} \times \text{length} \)

The base is a triangle, so the base area is the area of a triangle.

We can use variables to write a formula for the volume of a triangular prism.

For the triangular prism below:
The length of the prism is \( l \).
Each triangular face has base \( b \) and height \( h \).
The volume of the prism is:

\[
V = \text{base area} \times \text{length} \\
V = \frac{1}{2} bh \times l \\
V = \frac{1}{2} bhl
\]

**Example**

How much water can the water trough hold?
Give the answer in litres.
Solution

Capacity is the amount a container will hold, commonly measured in litres (L) or millilitres (mL). Volume is the amount of space an object occupies, commonly measured in cubic units.

The amount of water the trough can hold is the capacity of the triangular prism.

Sketch the prism.
Identify the variable that represents each dimension.
The base of the triangle is: \( b = 60 \)
The height of the triangle is: \( h = 40 \)
The length of the prism is: \( l = 120 \)
Substitute for each variable into the formula for volume.
\[
V = \frac{1}{2}bh
\]

\[
V = \frac{1}{2} \times 60 \times 40 \times 120
\]

\[
V = 144 \, 000
\]

The volume of the trough is 144,000 cm\(^3\).
1000 cm\(^3\) = 1 L
So, 144,000 cm\(^3\) = 144 L
The trough can hold 144 L of water.

Practice

1. The base area and length for each triangular prism are given. Find the volume of each prism.
   a) \( A = 9.2 \, \text{cm}^2 \) \( l = 2.3 \, \text{cm} \)
   b) \( A = 43.5 \, \text{cm}^2 \) \( l = 5 \, \text{cm} \)
   c) \( A = 3 \, \text{m}^2 \) \( l = 15 \, \text{m} \)

2. Find the volume of each triangular prism.
   a) \( h = 7 \, \text{cm} \) \( l = 21 \, \text{cm} \) \( w = 13 \, \text{cm} \)
   b) \( h = 8 \, \text{m} \) \( l = 5 \, \text{m} \) \( w = 12 \, \text{m} \)
   c) \( h = 1.75 \, \text{m} \) \( l = 2.50 \, \text{m} \) \( w = 1.75 \, \text{m} \)
3. Calculate the volume of each prism.
   a)  
   b)  

4. Find possible values for $b$, $h$, and $l$ for each volume of a triangular prism. Sketch one possible prism for each volume.
   a) $5 \text{ cm}^3$  
   b) $9 \text{ m}^3$  
   c) $8 \text{ m}^3$  
   d) $18 \text{ cm}^3$  

5. What is the volume of glass in this prism?

6. Any face can be used as the base of a rectangular prism.
   Can any face be used as the base of a triangular prism? Explain.

7. The volume of a triangular prism is $30 \text{ cm}^3$.
   Each triangular face has an area of $4 \text{ cm}^2$.
   How long is the prism?

8. a) Calculate the surface area and volume of this triangular prism.
   b) What do you think happens to the surface area and volume when the length of the prism is doubled? Justify your answer.
   Calculate the surface area and volume to check your ideas.
   c) What do you think happens to the surface area and volume when the base and height of the triangular faces are doubled?
   Justify your answer.
   Calculate the surface area and volume to check your ideas.
   d) What do you think happens to surface area and volume when all the dimensions are doubled?
   Justify your answer.
   Calculate the surface area and volume to check your ideas.
9. **Assessment Focus** Jackie uses this form to build a concrete pad.
   a) How much concrete will Jackie need to mix to fill the form?
   b) Suppose Jackie increases the equal sides of the form from 3 m to 6 m. How much more concrete will Jackie need to mix? Include a diagram.

10. A chocolate company produces different sizes of chocolate bars that are packaged in equilateral triangular prisms. Here is the 100-g chocolate bar.

   a) Calculate the surface area and volume of the box.
   b) The company produces a 400-g chocolate bar. It has the same shape as the 100-g bar.
      i) What are the possible dimensions for the 400-g box?
      ii) How are the dimensions of the two boxes related?

11. The volume and surface area of a prism, with a base that is not a triangle or a rectangle, can be found by dividing the prism into smaller prisms. Find the volume and surface area of each prism.

   a) 
   b) 

**Take It Further**

Describe the relationships among the dimensions, faces, and volume of a triangular prism. Include an example in your description.
Features of Word Problems

In Unit 2, you wrote problem statements as questions. Math problems include information to help you understand and solve the problem. Here are some features of math word problems.

- **Context** – may describe who, what, when, where, and why, like the setting of a story.

- **Math Information** – may include words, numbers, figures, drawings, tables, graphs, and/or models.

- **Problem Statement** – tells you to do something with the information. It may be a question or an instruction. Key words are sometimes used to suggest how the response is to be communicated.

Suppose you have an old refrigerator and are thinking about buying a new one.

The new refrigerator costs $900. It costs $126 a year to run your old fridge, and $66 a year to run the new, more energy-efficient fridge.

Is it more economical to buy the new fridge? Justify your answer.

To solve the problem, we need to compare the costs of buying the new fridge and running it with the costs of running the old fridge. We can use a table and a graph.

In the second column of the table, add $126 each year.
In the third column of the table, add $66 each year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Old Fridge Cost ($)</th>
<th>New Fridge Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>126</td>
<td>900 + 66 = 966</td>
</tr>
<tr>
<td>2</td>
<td>252</td>
<td>1032</td>
</tr>
<tr>
<td>3</td>
<td>378</td>
<td>1098</td>
</tr>
<tr>
<td>4</td>
<td>504</td>
<td>1164</td>
</tr>
<tr>
<td>5</td>
<td>630</td>
<td>1230</td>
</tr>
<tr>
<td>6</td>
<td>756</td>
<td>1296</td>
</tr>
<tr>
<td>7</td>
<td>882</td>
<td>1362</td>
</tr>
</tbody>
</table>

Here is part of the table and a graph.
Continue the table until the costs are equal.
Draw a graph for the data.
Use the table and graph to solve the problem.
With a partner, identify the context, math information, and problem statement for each of questions 1 to 4. Then solve the problem.

1. Hori is buying carpet for his living room. It is rectangular with dimensions 4 m by 5 m. How much will Hori save if he buys the carpet at the sale price shown at the left?

2. Suppose you are in charge of setting up the cafeteria for a graduation dinner. One hundred twenty-two people will attend. The tables seat either 8 or 10 people. You do not want empty seats. How many of each size table will you need to use to make sure everyone has a seat? List all the combinations.

3. To win a contest, you have to find a mystery number. The mystery number is described this way:

   Sixteen more than \( \frac{2}{3} \) of the mystery number is equal to two times the mystery number.

   What is the mystery number?

4. Alicia and Chantelle are playing a game. There are six tiles in a box: three red and three blue. A player picks two tiles without looking. Alicia gets a point if the tiles do not match; Chantelle gets a point if they do match. The tiles are returned to the box each time.

   What is the probability that both tiles have the same colour? Is this a fair game? Explain.

5. Write a word problem using the numbers 48, 149, and 600. Remember to include a context, math information, and a problem statement.

   Trade problems with a classmate. Identify the features of your classmate's problem. Solve the problem.
What Do I Need to Know?

☑ Euler's Formula
For any polyhedron, the numbers of faces, edges, and vertices are related by this formula:
vertices + faces − edges = 2

☑ Surface Area of a Triangular Prism
Surface area = sum of the areas of 3 rectangular faces +
2 × area of one triangular face
\[ SA = al + bl + cl + bh \]
The side lengths of a triangular face are \(a, b,\) and \(c\).
The height of a triangular face is \(h\).
The length of the prism is \(l\).

☑ Volume of a Triangular Prism
Volume = area of triangular base \(\times\) length of prism
\[ V = \frac{1}{2} bh l \]
The base and height of a triangular face are \(b\) and \(h\), respectively.
The length of the prism is \(l\).
What Should I Be Able to Do?

**LESSON**

1. You will need linking cubes and isometric dot paper.
   a) Build the object that matches the views below.
   b) Sketch the object on isometric dot paper.

2. Sketch as many objects as possible that have two different rectangles as two of its views.

3. This set of views represents an object.

4. Here is a net of a triangular prism.
   a) Calculate the surface area of the prism in square centimetres.
   b) Calculate the volume of the prism in cubic centimetres.

5. a) Calculate the surface area of this prism. Sketch a net first, if it helps.
   b) Calculate the volume of the prism.
6. The horticultural society is building a triangular flower bed at the intersection of two streets. The edges of the bed are raised 0.25 m. How much soil is needed to fill this flower bed? Justify your answer.

7. Find the possible values of \( b \), \( h \), and \( l \) for a triangular prism with volume 21 m\(^3\). How many different ways can you do this? Sketch a diagram of one possible prism.

8. Alijah volunteers with the horticultural society. He wants to increase the size but not the depth of the flower bed in question 6.
   a) How can Alijah change the dimensions so that:
      - the flower bed remains triangular, and
      - the area of the ground covered by the bed doubles?
   b) Sketch the new flower bed. Label its dimensions.
   c) How does the change in size affect the volume of soil needed? Explain.

9. The bucket on the front of a lawn tractor is a triangular prism.
   a) Find the volume of soil, in cubic metres, the bucket can hold. What assumptions do you make?
   b) Suppose the dimensions of the triangular faces are doubled. How much more soil do you expect the new bucket to hold? Explain.
   c) Calculate the new volume. Sketch the new bucket and include the new dimensions.

10. A tent has the shape of a triangular prism. Its volume is 25 m\(^3\).
    a) Find possible dimensions for this prism.
    b) Choose one set of dimensions from part a. Sketch the tent. Label the dimensions.
    c) A larger tent has volume 100 m\(^3\). It is also a triangular prism.
       i) What could the dimensions of this larger tent be? Justify your answer.
       ii) How are the dimensions of the two tents related?
1. You will need linking cubes and isometric dot paper.
   a) Build the object that has these views.
   b) Sketch the object on isometric dot paper.
   c) Draw a net for the object. Cut out and fold the net to make the object.

2. Calculate the surface area and volume of this prism.

3. Look at the triangular prism in question 2. Suppose the base and height of the triangular faces are tripled. 
   a) How does this affect the volume of the prism? Explain.
   b) Sketch the larger prism.
   c) Calculate the volume of the larger prism.

4. The triangular faces at the left are the bases of four triangular prisms. All the prisms have the same length.
   a) Which prism has the greatest volume? Explain.
   b) Which prism has the least surface area? Explain.

5. The volume of a triangular prism is 210 cm³.
   a) The length of the prism is 7 cm. What are the possible base and height of the triangular faces?
   b) On 1-cm grid paper, draw two possible triangular faces for this prism. Measure to find the lengths of any sides you do not know.
   c) Calculate the surface area of each prism whose face you drew in part b.
Suppose your class is responsible for building a circus tent. The organizers have given you the views below.

Work in a group.

**Part 1**

Prepare a presentation for the organizers. Your presentation must include:
- a 3-D sketch
- a net for the tent
- a list of steps for building the net
- a model of the tent
Part 2

Some members of a local service club will perform in the circus. They need a prop built for their act. It is a triangular prism in which one performer can hide, and which can also be used as a ramp for a bicycle jump. Prepare an estimated cost to build this prop.

Your estimate must include:
- a diagram of the prop with appropriate dimensions
- an explanation of the dimensions you chose
- calculations for the amount of materials used
- cost of materials if the building material is $16.50/m²

Part 3

The volume or capacity of the tent depends on its dimensions. Research situations where volume and capacity are used in your home.

Write a report on your findings.

Reflect on the Unit

Write a paragraph on what you have learned about representing three-dimensional objects and triangular prisms. Try to include something from each lesson in the unit.
Golden Rectangles

Work with a partner.

Similar polygons have the same shape, but different sizes. These two rectangles are similar.

![Rectangles](image)

Similar rectangles have the same base:height ratio. That is, 8 cm:4 cm = 4 cm:2 cm

In this *Investigation*, you will draw rectangles and compare the ratios of their side lengths. As you complete the *Investigation*, include all your work in a report that you will hand in.

**Part 1**

- Use 1-cm grid paper.
  - Turn your paper so the longer side is horizontal.
  - Draw a 1-cm square in the bottom left hand corner.
  - Label this square ABCD.
  - Draw another 1-cm square directly above the first.
  - You should now have a rectangle that measures 2 cm by 1 cm.
  - Label this rectangle ABEF.
  - Make a larger rectangle by drawing a square along a longer side of rectangle ABEF. Label this 3-cm by 2-cm rectangle AGHF.

- Continue to make larger rectangles by drawing squares along a longer side of the previous rectangle. Continue to label the rectangles with letters. Is there a pattern in the way you draw the rectangles? Explain.

- Stop drawing rectangles when you have no more room on the paper. Record the base and height of each rectangle in a table. For this *Investigation*, the base is always the longer side.
Write the ratio of the base to the height in fraction form. Calculate the value of the ratio by dividing the base by the height. Use a calculator if you need to. Round each number to 3 decimal places when necessary. Include these numbers in the table.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Base</th>
<th>Height</th>
<th>Ratio $\frac{b}{h}$</th>
<th>Ratio Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABCD</td>
<td>1</td>
<td>1</td>
<td>$\frac{1}{1}$</td>
<td>1.0</td>
</tr>
<tr>
<td>ABEF</td>
<td>2</td>
<td>1</td>
<td>$\frac{2}{1}$</td>
<td>2.0</td>
</tr>
</tbody>
</table>

What patterns do you see in the table?

Extend the patterns. What are the dimensions of the next 3 rectangles? How do you know?

Calculate the ratio of $\frac{b}{h}$ for each of the next 3 rectangles. What do you notice about the value of the ratio as the rectangles get larger?

Predict the value of the ratio for the 50th rectangle. What do you think the ratio will be for the 100th rectangle? Explain.

Are the rectangles in the table similar? Explain.

**Part 2**

Look for rectangles around the room. Some examples might be found in notebook paper, textbooks, and windows. Include these rectangles in your table.

Measure the base and height of each rectangle you include. What is the ratio of $\frac{b}{h}$ for each? What is the value of this ratio?

**Take It Further**

Find some pictures of structures or buildings. Look for rectangles in these structures. Find the values of $\frac{b}{h}$ for each rectangle. Does any rectangle have the same ratio as the last rectangle in your table?