Look at the figures at the right. How is each figure made from the previous figure? The equilateral triangle in Step 1 has side length 1 unit. What is the perimeter of each figure?

What patterns do you see? Suppose the patterns continue. What will the perimeter of the figure in Step 4 be? For any figure after Step 1, how is its perimeter related to the preceding figure?

What You’ll Learn

- Compare and order fractions.
- Add and subtract fractions.
- Multiply a fraction by a whole number and by a fraction.
- Divide a whole number by a fraction, and a fraction by a fraction.
- Convert between fractions and decimals.

Why It’s Important

You use fractions and decimals when you shop, measure, and work with a percent; and in sports, recipes, and business.
Key Words

- reciprocals
- terminating decimal
- repeating decimal

Step 2

Step 3
Multiply a Fraction by a Whole Number

We multiply a fraction by a whole number to find that fraction of the whole number.

Example
a) Find \( \frac{1}{3} \) of 27.

Solution
a) \( \frac{1}{3} \) of 27 is
\[
\frac{1}{3} \times 27 = 27 \times \frac{1}{3}
\]
Think: 27 times \( \frac{1}{3} \) is 27 thirds.
\[
\frac{1}{3} \times 27 = \frac{27}{3} = 9
\]

b) Multiply \( \frac{3}{4} \times 6 \)

\[
\frac{3}{4} \times 6 = \frac{18}{4} = \frac{9}{2}
\]

In Example part a, note that \( 27 \times \frac{1}{3} \) is the same as \( 27 \div 3 \).
To find \( \frac{1}{3} \) of a number, we multiply by \( \frac{1}{3} \) or divide by 3.
\( \frac{1}{3} \) and 3 are reciprocals.
To multiply a number by a unit fraction, we can divide by the reciprocal instead.

Check

1. Find.
   a) \( \frac{1}{5} \) of 25
   b) \( \frac{1}{4} \) of 64
   c) \( \frac{1}{8} \) of 40

2. Multiply.
   a) \( \frac{3}{2} \times 20 \)
   b) \( \frac{5}{3} \times 5 \)
   c) \( \frac{7}{9} \times 4 \)

3. There are 660 students in Parkside School from Kindergarten to Grade 8.
   a) Three-quarters of the students are boys.
   How many boys attend the school?
   b) One-third of the students are in K to Grade 4.
   How many students are in Grades 5 to 8?
Some photographers use a manual camera with a shutter speed dial. The numbers on the dial show how long the shutter stays open when a person takes a picture.

This setting opens the shutter for 2 s.

This setting opens the shutter for $\frac{1}{2}$ of 1 s.

This setting opens the shutter for $\frac{1}{4}$ of 1 s.

This pattern continues.
How long is the shutter open when the setting is 8? 15? 30? Does a setting of 8 allow more or less light than a setting of 15? Suppose a setting of 125 does not allow enough light. Which setting might allow enough light?

**Explore**

Work in a group.

This square has side length 1 unit.

Your teacher will give you a copy of this square. What fraction of the whole square is each piece? Order the fractions from least to greatest.

**Reflect & Share**

Share your results with those of another group of classmates. How did you write each piece as a fraction of the whole? What strategies did you use to order the fractions? Could you use these strategies to order any set of fractions? Explain.
One way to compare fractions is to use equivalent fractions. Write each fraction with the same denominator, then compare the numerators.

For example, to order $\frac{5}{8}$, $\frac{4}{5}$, and $\frac{3}{4}$:
Write equivalent fractions for each fraction until all the fractions have the same denominator:

$$
\frac{5}{8} = \frac{10}{16} = \frac{15}{24} = \frac{20}{32} = \frac{25}{40}
$$

$$
\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} = \frac{28}{35} = \frac{32}{40}
$$

$$
\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \frac{21}{28} = \frac{24}{32} = \frac{27}{36} = \frac{30}{40}
$$

Now, each fraction has denominator 40.
Compare the numerators: $25 < 30 < 32$
So, $\frac{25}{40} < \frac{30}{40} < \frac{32}{40}$
So, $\frac{5}{8} < \frac{3}{4} < \frac{4}{5}$

In order from least to greatest: $\frac{5}{8}, \frac{3}{4}, \frac{4}{5}$
A simpler way to find the common denominator is to find the lowest common multiple of the denominators.
We can use this method to order improper fractions.

**Example**

Write these fractions in order from least to greatest:
$\frac{5}{3}, \frac{3}{2}, \frac{8}{5}$

**Solution**

In an improper fraction, the numerator is greater than the denominator.

Find the lowest common denominator.
Since the denominators have no common factors, list the multiples of 3, 2, and 5.
3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...
2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, ...
5: 5, 10, 15, 20, 25, 30, ...
The lowest common denominator is 30.

\[
\begin{align*}
\frac{5}{3} \times 10 &= \frac{50}{30} \\
\frac{3}{2} \times 15 &= \frac{45}{30} \\
\frac{8}{5} \times 6 &= \frac{48}{30}
\end{align*}
\]

So, in order from least to greatest: \(\frac{3}{2}, \frac{8}{5}, \frac{5}{3}\)

Use this method to find the lowest common denominator when the denominators have no common factors.

In the Example, notice that the lowest common multiple of 3, 2, and 5 is their product: \(3 \times 2 \times 5 = 30\)

When two or more numbers have no common factors, their lowest common multiple is their product.

**Practice**

1. In each pair, which fraction is greater? How do you know?
   
   a) \(\frac{1}{2}, \frac{2}{5}\)  
   b) \(\frac{2}{3}, \frac{5}{6}\)  
   c) \(\frac{1}{2}, \frac{2}{3}\)  
   d) \(\frac{3}{4}, \frac{2}{5}\)
   
   e) \(\frac{1}{4}, \frac{1}{3}\)  
   f) \(\frac{2}{3}, \frac{3}{4}\)  
   g) \(\frac{3}{4}, \frac{5}{8}\)  
   h) \(\frac{2}{3}, \frac{3}{10}\)

2. Order the fractions in each set from least to greatest.
   
   a) \(\frac{3}{8}, \frac{4}{5}, \frac{1}{2}\)  
   b) \(\frac{7}{10}, \frac{6}{8}, \frac{3}{5}\)  
   c) \(\frac{5}{2}, \frac{6}{3}, \frac{7}{4}\)  
   d) \(\frac{10}{3}, \frac{7}{5}, \frac{13}{6}\)

3. Use the fractions \(\frac{19}{10}, \frac{11}{3}, \frac{9}{4}\).
   
   a) Order the fractions from least to greatest.
   b) Write each fraction as a mixed number.
   c) Order the mixed numbers from least to greatest.
   d) Which method was easier: ordering the improper fractions or ordering the mixed numbers? Explain.

   When would you use the method of ordering mixed numbers?

4. Maria stated that \(\frac{5}{6}\) is between \(\frac{4}{5}\) and \(\frac{6}{7}\).
   Do you agree? Give reasons for your answer.

5. Find the fraction that is halfway between each pair of numbers.
   Use a number line if it helps.
   
   a) 0 and 1  
   b) 1 and 2  
   c) 0 and \(\frac{1}{2}\)  
   d) \(\frac{1}{2}\) and 1  
   e) 1 and \(\frac{3}{2}\)  
   f) \(\frac{3}{2}\) and 2
The Farey sequence is named for the British geologist and mathematician, John Farey, who lived from 1766 to 1826.

6. A Farey sequence is a list of certain fractions that follow a pattern.

1st Farey sequence: \( \frac{0}{1}, \frac{1}{1} \)

2nd Farey sequence: \( \frac{0}{1}, \frac{1}{1}, \frac{1}{2} \)

3rd Farey sequence: \( \frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3} \)

4th Farey sequence: \( \frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{1}{3}, \frac{3}{4} \)

a) Look at the lists above. Extend the pattern.
Write the 5th Farey sequence.

b) Order the fractions in the 5th Farey sequence from least to greatest.

7. **Assessment Focus** The fraction \( \frac{11}{2} \) is halfway between 5 and 6.
The fraction \( \frac{23}{4} \) is halfway between \( \frac{11}{2} \) and 6.

a) How many more fractions can you find between 5 and 6?
List all the fractions you find.

b) Have you found all the fractions between 5 and 6?
How do you know?
Show your work.

8. a) Use the digits 3, 4, 5, and 6 to write as many proper and improper fractions as you can.
Each numerator and denominator is a single digit.

b) Order the fractions in part a from least to greatest.

c) Which fractions in part a are:
   i) less than \( \frac{1}{2} \)?
   ii) between \( \frac{1}{2} \) and 1?
   iii) greater than 1?

9. In each pair, which fraction is greater? How do you know?

a) \( \frac{22}{32} \) or \( \frac{43}{65} \)

b) \( \frac{91}{99} \) or \( \frac{919}{999} \)

Take It Further

**Reflect**

Name two ways you can compare and order fractions.
Which way do you prefer? Explain.
Explore

Work on your own.
Copy these diagrams.

\[ \square + \square = \square + \square = \]

Use the numbers 1, 2, 4, and 8 to make the greatest sum and the least sum.
In each case, use each number once.

Reflect & Share
Share your results with a classmate.
Do both of you have the same answers?
If not, which is the greatest sum? The least sum?
What strategies did you use to add?

Connect

Recall how to add fractions with the same denominator.
For example, to add \( \frac{3}{12} \) and \( \frac{4}{12} \),
add the numerators: \( \frac{3}{12} + \frac{4}{12} = \frac{7}{12} \)
We can illustrate this sum with a diagram.

To add fractions that do not have the same denominator, we find and use a common denominator.
Example 1

Add. \( \frac{5}{12} + \frac{5}{6} \)

Solution

\[ \frac{5}{12} + \frac{5}{6} \]

Use equivalent fractions to write the fractions with a common denominator.
Since 6 is a factor of 12, the lowest common multiple of 12 and 6 is 12.

Use 12 as the common denominator.

\[
\begin{align*}
\times 2 & \quad \times 2 \quad \times 2 \\
\frac{5}{6} & = \frac{10}{12} \\
\frac{5}{12} + \frac{5}{6} & = \frac{5}{12} + \frac{10}{12} \\
& = \frac{15}{12} \\
& = \frac{15}{12} + \frac{3}{3} \\
& = \frac{5}{4}
\end{align*}
\]

A fraction is in simplest form when the numerator and denominator have no common factors.

Add the numerators.
To reduce to simplest form, divide the numerator and denominator by their greatest common factor, 3.
Since 5 > 4, this is an improper fraction.

To write the fraction as a mixed number:

\[
\frac{5}{4} = \frac{4}{4} + \frac{1}{4} = 1 + \frac{1}{4} = 1 \frac{1}{4}
\]

This is a mixed number.

We can also use common denominators to add more than two fractions.

Example 2

Add. \( \frac{2}{3} + \frac{4}{5} + \frac{3}{4} \)

Solution

\[ \frac{2}{3} + \frac{4}{5} + \frac{3}{4} \]

Use equivalent fractions to write the fractions with a common denominator.
The denominators 3, 5, and 4 have no common factors.  
So, their lowest common multiple is their product: \(3 \times 5 \times 4 = 60\)  
Write each fraction with denominator 60.

\[
\begin{align*}
\frac{2}{3} \times 20 &= \frac{40}{60} \\
\frac{4}{5} \times 12 &= \frac{48}{60} \\
\frac{3}{4} \times 15 &= \frac{45}{60}
\end{align*}
\]

\[
\frac{2}{3} + \frac{4}{5} + \frac{3}{4} = \frac{40}{60} + \frac{48}{60} + \frac{45}{60} = \frac{133}{60}
\]

Since 133 > 60, this is an improper fraction.  
It can be written as a mixed number.

\[
= \frac{120}{60} + \frac{13}{60} = 2 + \frac{13}{60} = 2 \frac{13}{60}
\]

**Practice**

Write all sums in simplest form.

1. Add.
   - a) \(\frac{4}{9} + \frac{1}{3}\)
   - b) \(\frac{1}{2} + \frac{1}{3}\)
   - c) \(\frac{2}{3} + \frac{1}{6}\)
   - d) \(\frac{3}{4} + \frac{1}{6}\)
   - e) \(\frac{2}{5} + \frac{1}{3}\)
   - f) \(\frac{2}{5} + \frac{1}{10}\)
   - g) \(\frac{1}{12} + \frac{1}{4}\)
   - h) \(\frac{3}{8} + \frac{1}{4}\)

2. Add.
   - a) \(\frac{3}{8} + \frac{3}{2}\)
   - b) \(\frac{7}{4} + \frac{4}{5}\)
   - c) \(\frac{7}{6} + \frac{5}{7}\)
   - d) \(\frac{13}{10} + \frac{4}{3}\)
   - e) \(\frac{5}{8} + \frac{2}{3}\)
   - f) \(\frac{4}{5} + \frac{4}{7}\)
   - g) \(\frac{9}{4} + \frac{4}{9}\)
   - h) \(\frac{8}{5} + \frac{11}{6}\)

3. Damara and Baldwin had to shovel snow from their driveway.  
   Damara shovelled about \(\frac{3}{10}\) of the driveway.  
   Baldwin shovelled about \(\frac{2}{3}\) of the driveway.  
   About what fraction of the driveway was cleared of snow?

4. a) Write each fraction as the sum of two fractions  
   with the same denominator.
   - i) \(\frac{1}{2}\)
   - ii) \(\frac{3}{4}\)
   - iii) \(\frac{9}{10}\)
   
   b) Write each fraction in part a as the sum of two fractions  
   with different denominators.
5. **Assessment Focus** Write each fraction as the sum of two or more fractions in as many different ways as you can.
   
   a) \( \frac{4}{5} \)  
   b) \( \frac{7}{10} \)  
   c) \( \frac{2}{9} \)

   Show your work.

6. Add.
   
   a) \( 3\frac{1}{3} + 4\frac{1}{4} \)  
   b) \( 2\frac{1}{2} + 1\frac{9}{10} \)  
   c) \( 1\frac{3}{4} + 2\frac{3}{5} \)  
   d) \( \frac{7}{8} + 1\frac{2}{3} \)  
   e) \( 2\frac{3}{5} + \frac{2}{3} \)  
   f) \( 5\frac{2}{5} + 1\frac{7}{8} \)  

7. Two students, Galen and Mai, worked on a project.
   Galen worked for \( 3\frac{2}{3} \) h.
   Mai worked for \( 5\frac{4}{5} \) h.
   What was the total time spent on the project?

8. Add.
   
   a) \( \frac{1}{2} + \frac{2}{3} + \frac{3}{4} \)  
   b) \( \frac{1}{3} + \frac{3}{4} + \frac{2}{5} \)  
   c) \( \frac{5}{6} + \frac{4}{5} + \frac{4}{3} \)  
   d) \( \frac{5}{4} + \frac{3}{5} + \frac{1}{6} \)  
   e) \( \frac{7}{10} + \frac{7}{5} + \frac{7}{2} \)  
   f) \( \frac{5}{12} + \frac{6}{5} + \frac{3}{4} \)  

9. Each fraction below is written as the sum of two unit fractions.
   Which sums are correct? How do you know?
   
   a) \( \frac{7}{10} = \frac{1}{5} + \frac{1}{2} \)  
   b) \( \frac{5}{12} = \frac{1}{3} + \frac{1}{4} \)  
   c) \( \frac{5}{6} = \frac{1}{3} + \frac{1}{3} \)  
   d) \( \frac{7}{12} = \frac{1}{2} + \frac{1}{6} \)  
   e) \( \frac{11}{8} = \frac{1}{2} + \frac{1}{9} \)  
   f) \( \frac{2}{15} = \frac{1}{10} + \frac{1}{30} \)  
   g) \( \frac{7}{15} = \frac{1}{5} + \frac{1}{3} \)  
   h) \( \frac{2}{5} = \frac{1}{3} + \frac{1}{15} \)  
   i) \( \frac{4}{15} = \frac{1}{5} + \frac{1}{15} \)  

10. Write each fraction as the sum of two different unit fractions.
    
    a) \( \frac{3}{4} \)  
    b) \( \frac{5}{12} \)  
    c) \( \frac{7}{10} \)  

11. Find this sum. Explain your method.
    
    \( \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{2}{3} + \frac{2}{4} + \frac{2}{5} + \frac{2}{6} + \frac{3}{4} + \frac{3}{5} + \frac{3}{6} + \frac{4}{5} + \frac{4}{6} + \frac{5}{6} \)  

[Image: Reflect]

Choose two improper fractions. Add them.
Write each improper fraction as a mixed number.
Add the mixed numbers.
Which method is more efficient for finding the sum of two improper fractions? Why?
Explore

Work with a partner.
You will need 1-cm grid paper and coloured pencils.

Use these rules to create a rectangular design on grid paper:
- The design must have line symmetry or rotational symmetry.
- One-half of the grid squares must be red.
  One-third of the grid squares must be blue.
  The remaining grid squares must be green.
- The rectangle must have the fewest squares possible.

What fraction of the squares are green? How do you know?
How many squares did you use? Explain.
Describe your design.

Reflect & Share

Compare your design with that of another pair of classmates.
If the designs are different, do both of them obey the rules?
Explain.
Compare your designs with those of other classmates.
How many different designs are possible?

Connect

To subtract fractions, we use a strategy similar to that for adding fractions.
When the denominators are different, we find a common denominator first.
Example 1

Subtract. \( \frac{4}{5} - \frac{3}{10} \)

**Solution**

\[
\frac{4}{5} - \frac{3}{10}
\]

Since 5 is a factor of 10, the lowest common denominator is 10.

\[
\begin{align*}
\frac{4}{5} \times 2 &= \frac{8}{10} \\
\end{align*}
\]

\[
\frac{4}{5} - \frac{3}{10} = \frac{8}{10} - \frac{3}{10}
\]

\[
= \frac{5}{10}
\]

\[
= \frac{5 \div 5}{10 \div 5}
\]

\[
= \frac{1}{2}
\]

This is not in simplest form.

5 is a factor of the numerator and denominator.

To subtract mixed numbers, we subtract the fractions, then subtract the whole numbers. We must check the fractions to see which is greater. When the second fraction is greater than the first fraction, we cannot subtract directly.

Example 2

Subtract. \( 3 \frac{1}{5} - 1 \frac{3}{4} \)

**Solution**

**Method 1**

Subtract the whole numbers and the fractions separately.

\( 3 \frac{1}{5} - 1 \frac{3}{4} \)

Subtract the fractions: \( \frac{1}{5} - \frac{3}{4} \)

But \( \frac{1}{5} < \frac{3}{4} \), so we cannot subtract \( \frac{3}{4} \) from \( \frac{1}{5} \).

Write \( 3 \frac{1}{5} \) as \( 2 + 1 \frac{1}{5} \), or \( 2 + \frac{6}{5} \).

The problem can be written \( 2 \frac{6}{5} - 1 \frac{3}{4} \).

Then, subtract the fractions.

\[
\frac{6}{5} - \frac{3}{4}
\]

The denominators 5 and 4 have no common factors.
So, their lowest common denominator is \(4 \times 5 = 20\).

\[
\begin{align*}
\frac{6}{5} \times 4 &= \frac{24}{20} \\
\frac{3}{4} \times 5 &= \frac{15}{20}
\end{align*}
\]

\[
\frac{6}{5} - \frac{3}{4} = \frac{24}{20} - \frac{15}{20} = \frac{9}{20}
\]

Now, subtract the whole numbers that remain: \(2 - 1 = 1\).

So, \(3\frac{1}{5} - 1\frac{3}{4} = 1\frac{9}{20}\)

**Method 2**

Change both fractions to improper fractions, then subtract.

\[
\begin{align*}
3\frac{1}{5} &= 3 + \frac{1}{5} = \frac{15}{5} + \frac{1}{5} = \frac{16}{5} \\
1\frac{3}{4} &= 1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}
\end{align*}
\]

The denominators have no common factors.

So, their lowest common denominator is \(4 \times 5 = 20\).

\[
\begin{align*}
\times 4 & \quad \times 5 \\
\frac{16}{5} &= \frac{64}{20} \\
\frac{7}{4} &= \frac{35}{20}
\end{align*}
\]

\[
\frac{16}{5} - \frac{7}{4} = \frac{64}{20} - \frac{35}{20} = \frac{29}{20}
\]

To write the fraction as a mixed number:

\[
\frac{29}{20} = \frac{20}{20} + \frac{9}{20} = 1 + \frac{9}{20} = 1\frac{9}{20}
\]

So, \(3\frac{1}{5} - 1\frac{3}{4} = 1\frac{9}{20}\)

In *Example 2*, the answer was written as a mixed number because the question was written with mixed numbers.

In general, the answer should be written in the same form as the question.
1. Subtract.
   a) \(\frac{7}{2} - \frac{5}{4}\)  
   b) \(\frac{7}{8} - \frac{3}{4}\)  
   c) \(\frac{13}{6} - \frac{8}{12}\)  
   d) \(\frac{5}{3} - \frac{2}{6}\)  
   e) \(\frac{7}{5} - \frac{4}{10}\)  
   f) \(\frac{5}{3} - \frac{2}{9}\)  
   g) \(\frac{7}{2} - \frac{2}{4}\)  
   h) \(\frac{3}{2} - \frac{9}{7}\)  

2. Subtract.
   a) \(\frac{11}{12} - \frac{5}{6}\)  
   b) \(\frac{7}{10} - \frac{1}{2}\)  
   c) \(\frac{3}{4} - \frac{3}{5}\)  
   d) \(\frac{7}{8} - \frac{1}{3}\)  
   e) \(\frac{2}{3} - \frac{7}{12}\)  
   f) \(\frac{7}{5} - \frac{2}{3}\)  
   g) \(\frac{9}{5} - \frac{1}{2}\)  
   h) \(\frac{4}{5} - \frac{1}{3}\)  

3. A sports store placed an order for shoes. 
   Three-eighths of the order was basketball shoes; 
   one-quarter was running shoes; and the rest were golf shoes. 
   What fraction of the order was golf shoes? 

4. Subtract.
   a) \(\frac{10}{3} - \frac{3}{4}\)  
   b) \(\frac{8}{5} - \frac{2}{3}\)  
   c) \(\frac{7}{4} - \frac{3}{5}\)  
   d) \(\frac{17}{10} - \frac{5}{6}\)  
   e) \(\frac{7}{2} - \frac{3}{5}\)  
   f) \(\frac{13}{6} - \frac{2}{5}\)  
   g) \(\frac{7}{3} - \frac{3}{2}\)  
   h) \(\frac{7}{3} - \frac{5}{8}\)  

5. Assessment Focus  
   Copy this diagram. 
   Use the numbers 2, 3, 4, and 5 as numerators or denominators. 

   \[
   \frac{\square}{\square} - \frac{\square}{\square} =
   \]

   a) Write as many different subtraction statements as possible. 
   Use each number once each time. 
   b) Which statement has the greatest difference? 
   c) Which statement has the least difference? 
   Show your work. 

   a) \(3\frac{3}{4} - 1\frac{1}{5}\)  
   b) \(3\frac{2}{5} - 1\frac{5}{8}\)  
   c) \(3\frac{7}{10} - 2\frac{1}{3}\)  
   d) \(3\frac{1}{3} - 2\frac{7}{10}\)  
   e) \(4\frac{2}{9} - 1\frac{1}{6}\)  
   f) \(4\frac{1}{6} - 1\frac{2}{5}\)  

7. a) Subtract.
   i) \(3 - \frac{4}{5}\)  
   ii) \(4 - \frac{3}{7}\)  
   iii) \(5 - \frac{5}{6}\)  
   b) Which methods did you use in part a? Explain your choice.
8. Write each fraction as the difference of two proper fractions with different denominators.
   a) $\frac{1}{2}$  b) $\frac{3}{4}$  c) $\frac{1}{10}$  d) $\frac{1}{6}$  e) $\frac{1}{4}$

9. a) One-half of the books in Kevin’s backpack are novels.
   He also has 3 science books, 2 history books, and 1 geography book.
   How many books are in Kevin’s backpack?
   b) In Raji’s locker, one-third of the books are novels and one-third are science books. She also has 2 geography books, 3 history books, and 1 social studies book.
   How many books are in Raji’s locker?

10. Here are some subtraction statements with unit fractions having denominators that are consecutive whole numbers:
    \[ \frac{1}{1} - \frac{1}{2}; \frac{1}{2} - \frac{1}{3}; \frac{1}{3} - \frac{1}{4}; \frac{1}{4} - \frac{1}{5} \]
    a) Find the difference of the fractions in each pair above.
    b) Write more pairs of fractions like these.
       Find each difference.
    c) What patterns do you see?

11. There are some pennies on a table.
    One-quarter of the pennies show heads.
    Two pennies are turned over.
    Now, one-third of the pennies show heads.
    How many pennies are on the table?
    How do you know?

12. Two matching pitchers contain grapefruit juice.
    Pitcher A is $\frac{1}{3}$ full. Pitcher B is $\frac{2}{3}$ full.
    Each pitcher is then filled with water.
    The contents of both pitchers are poured into one large bowl.
    What fraction of the liquid is grapefruit juice?

Take It Further

Reflect

Which fractions or mixed numbers are easy to subtract?
Which are more difficult?
Give an example in each case.
You have used area models to multiply 2 whole numbers, and to multiply a whole number and a fraction. We extend the area model to multiply 2 fractions.

**Explore**

Work with a partner.
One-quarter of a cherry pie was left after dinner.
Trevor ate one-half of the leftover pie for lunch the next day.
What fraction of the pie did he have for lunch?
What if Trevor had eaten only one-quarter of the leftover pie.
What fraction of the pie would he have eaten?

**Reflect & Share**

How did you solve the problems?
Compare your solutions and strategies with those of another pair of classmates.
Was one strategy more efficient than another? Explain.

**Connect**

Use an area model to find the product of fractions.
For example:
Sandi cut \( \frac{2}{3} \) of the grass on a rectangular lawn.
Akiva cut \( \frac{1}{2} \) of the remaining grass.
What fraction of the lawn did Akiva cut?

Sandi cut \( \frac{2}{3} \). So, \( \frac{1}{3} \) remains to be cut.
Akiva cut \( \frac{1}{2} \) of \( \frac{1}{3} \).
From the rectangle, $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$
$\frac{1}{6}$ is the area of part of a rectangle, with one side length $\frac{1}{2}$ of the original length and the other side length $\frac{1}{3}$ of the original length.
Since the area of a rectangle is length $\times$ width, we can write $\frac{1}{2}$ of $\frac{1}{3}$ as the multiplication statement: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
So, Akiva cut $\frac{1}{6}$ of the lawn.

**Example**

One-half of the Grade 8 students tried out for the school’s volleyball team. Three-quarters of the students were successful.

a) What fraction of the Grade 8 students are on the team?
b) Draw an area model and write a multiplication statement to show your answer.

**Solution**

a) Three-quarters of one-half of the Grade 8 students are on the team.

Draw a rectangle.
Show $\frac{1}{2}$ of the rectangle.

Divide $\frac{1}{2}$ of the rectangle into quarters.
Shade $\frac{3}{4}$.

Use broken lines to divide the whole rectangle into equal parts.
There are 8 equal parts.
Three parts are shaded.
So, $\frac{3}{8}$ of the Grade 8 students are on the team.

b) $\frac{3}{4}$ of $\frac{1}{2}$ is $\frac{3}{8}$.
So, $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
1. Draw each rectangle on grid paper. Use the rectangle to find each product.
   a) \( \frac{1}{2} \times \frac{3}{4} \)
   b) \( \frac{3}{4} \times \frac{2}{3} \)
   c) \( \frac{2}{5} \times \frac{1}{2} \)
   d) \( \frac{5}{6} \times \frac{1}{2} \)
   e) \( \frac{3}{5} \times \frac{7}{8} \)
   f) \( \frac{4}{5} \times \frac{3}{4} \)

2. Draw a rectangle on grid paper to find each product.
   a) \( \frac{3}{4} \times \frac{5}{8} \)
   b) \( \frac{4}{9} \times \frac{2}{5} \)
   c) \( \frac{3}{4} \times \frac{2}{3} \)
   d) \( \frac{6}{7} \times \frac{2}{3} \)
   e) \( \frac{2}{3} \times \frac{1}{3} \)
   f) \( \frac{4}{5} \times \frac{4}{5} \)

3. Write 3 multiplication statements using proper fractions. Make sure they are different from any products you have found so far. Draw a rectangle to illustrate each product.

4. **Assessment Focus**
   a) Draw an area model to find each product.
      i) \( \frac{3}{4} \times \frac{2}{5} \)
      ii) \( \frac{2}{4} \times \frac{3}{5} \)
      iii) \( \frac{1}{4} \times \frac{3}{8} \)
      iv) \( \frac{3}{4} \times \frac{1}{8} \)
      v) \( \frac{3}{5} \times \frac{4}{6} \)
      vi) \( \frac{3}{6} \times \frac{4}{5} \)
   b) Compare the area models. What patterns do you see? Write some other products similar to those in part a. Show your work.

5. Why is \( \frac{5}{8} \) of \( \frac{3}{12} \) equal to \( \frac{3}{8} \) of \( \frac{5}{12} \)?
   Draw area models to explain your answer.

---

**Reflect**

When you use an area model to multiply two fractions, how do you decide how to draw the rectangle?
Include an example in your explanation.
Explore

Work with a partner.
You will need grid paper.
Copy this diagram on grid paper.

Find \( \frac{2}{3} \times \frac{4}{5} \).
What patterns do you notice in the numbers?
How can you use patterns to multiply \( \frac{2}{3} \times \frac{4}{5} \)?
Use your method to calculate \( \frac{7}{8} \times \frac{3}{10} \).
Use an area model to check.

Reflect & Share

Compare your strategies with those of another pair of classmates.
Do you think your strategy will work with all fractions? Explain.

Connect

Here is an area model to show:
\[
\frac{4}{7} \times \frac{2}{5} = \frac{8}{35}
\]

The product of the numerators is:
\[4 \times 2 = 8\]
The product of the denominators is:
\[7 \times 5 = 35\]
That is, \( \frac{4}{7} \times \frac{2}{5} = \frac{4 \times 2}{7 \times 5} = \frac{8}{35} \)

So, to multiply two fractions, multiply the numerators and multiply the denominators.

We can use this method to multiply proper fractions and improper fractions.

*Example 1* illustrates that the product expression may be simplified before multiplying.
Example 1

Multiply.

a) \( \frac{4}{9} \times \frac{3}{8} \)

Solution

a) \( \frac{4}{9} \times \frac{3}{8} = \frac{4 \times 3}{9 \times 8} \)

Notice that the numerator and denominator have common factors 3 and 4.

Divide the numerator and denominator by these factors.

\[
\frac{4}{9} \times \frac{3}{8} = \frac{\frac{4}{3}}{\frac{9}{2}} = \frac{1}{3 \times 2} = \frac{1}{6}
\]

b) \( \frac{7}{5} \times \frac{8}{3} \)

There are no common factors in the numerators and denominators.

So, \( \frac{7}{5} \times \frac{8}{3} = \frac{56}{15} \)

Here is an area model to illustrate the product of the improper fractions in Example 1b.
Each small square is $\frac{1}{15}$.
The number of shaded squares is: $15 + 15 + 10 + 6 + 6 + 4 = 56$
So, $\frac{56}{15}$ are shaded.
$\frac{7}{5} \times \frac{8}{3} = \frac{56}{15}$

To multiply two mixed numbers, change them to improper fractions first.

**Example 2**

Multiply, $2\frac{1}{4} \times 3\frac{2}{5}$

**Solution**

\[
2\frac{1}{4} \times 3\frac{2}{5} = \frac{9}{4} \times \frac{17}{5}
\]

As a mixed number: $\frac{153}{20} = 7\frac{13}{20}$

So, $2\frac{1}{4} \times 3\frac{2}{5} = \frac{153}{20}$, or $7\frac{13}{20}$

We can apply the rules for multiplying two fractions to multiply three or more fractions.
You will do this in Practice question 7.

**Practice**

1. Multiply. Check each product by drawing an area model.
   a) $\frac{3}{8} \times \frac{5}{6}$
   b) $\frac{4}{5} \times \frac{1}{2}$
   c) $\frac{3}{10} \times \frac{3}{4}$

2. Multiply.
   a) $\frac{3}{5} \times \frac{2}{3}$
   b) $\frac{1}{2} \times \frac{5}{10}$
   c) $\frac{1}{6} \times \frac{1}{4}$
   d) $\frac{13}{8} \times \frac{3}{2}$
   e) $\frac{5}{4} \times \frac{11}{10}$
   f) $\frac{7}{3} \times \frac{7}{8}$
3. Paula has \( \frac{7}{8} \) of a tank of gas.
She estimates she will use \( \frac{2}{3} \) of the gas to get home.
What fraction of a tank of gas does she use?

4. a) Find each product.
   i) \( \frac{3}{4} \times \frac{4}{3} \)  
   ii) \( \frac{1}{5} \times \frac{5}{1} \)
   iii) \( \frac{7}{2} \times \frac{2}{7} \)  
   iv) \( \frac{5}{6} \times \frac{6}{5} \)
   v) \( \frac{8}{3} \times \frac{3}{8} \)  
   vi) \( \frac{12}{11} \times \frac{11}{12} \)

b) What do you notice about the products in part a?
Write 3 more pairs of fractions that have
the same product.
What is special about these fractions?

5. Multiply.
   a) \( 1\frac{3}{4} \times 2\frac{1}{2} \)  
   b) \( 3\frac{3}{5} \times 2\frac{1}{5} \)  
   c) \( 4\frac{3}{8} \times 1\frac{1}{4} \)
   d) \( 3\frac{3}{4} \times 3\frac{3}{4} \)  
   e) \( 4\frac{3}{10} \times \frac{4}{5} \)  
   f) \( \frac{7}{8} \times 2\frac{3}{5} \)

6. Play this game with a partner.
   Your teacher will give you a copy of this spinner.

You will need an open paperclip as a pointer and
a sharp pencil to keep the pointer in place.
For each turn, players spin twice.
Player A adds the fractions.
Player B multiplies the same two fractions.
The player with the greater result gets one point.
The first person to get 12 points wins.
Is this game fair?
Give reasons for your answer.
7. Use your knowledge of exponents and multiplying fractions to evaluate each power.
   a) \( \left( \frac{2}{5} \right)^{2} \)  
   b) \( \left( \frac{6}{5} \right)^{3} \)  
   c) \( \left( \frac{3}{10} \right)^{4} \)  
   d) \( \left( \frac{5}{2} \right)^{4} \)

8. **Assessment Focus** In question 4, each product is 1.
   a) Write a pair of fractions that has each product.
      i) 2  ii) 3  iii) 4  iv) 5
   b) Write a pair of fractions that has the product 1.
      Change only one numerator or denominator each time to write a pair of fractions that has each product.
      i) 2  ii) 3  iii) 4  iv) 5
   c) How could you write a pair of fractions that has the product 10?
      Show your work.

9. The product of two fractions is \( \frac{2}{3} \).
   One fraction is \( \frac{3}{5} \).
   What is the other fraction?
   How do you know?

10. Amar baked a cake. John ate \( \frac{1}{6} \) of the cake.
    Susan ate \( \frac{1}{5} \) of what was left.
    Chan ate \( \frac{1}{4} \) of what was left after that.
    Cindy ate \( \frac{1}{3} \) of what was left after that.
    Luigi ate \( \frac{1}{2} \) of what was left after that.
    How much of the original cake was left?

**Take It Further**

**Reflect**

Look at your answers to all the questions.
Some products were in simplest form after you multiplied.
Some products were not in simplest form.
How can you tell if a product of two fractions will be in simplest form after you multiply? Use examples in your explanation.
1. Order these fractions from least to greatest.
   \[
   \frac{2}{3}, \frac{1}{2}, \frac{5}{8}, \frac{1}{4}, \frac{3}{4}
   \]

2. Paola has read \(\frac{3}{4}\) of her novel. Rafferty has read \(\frac{5}{7}\) of the same novel.
   Who has read more?
   How do you know?

3. Which fractions below are:
   a) between 0 and \(\frac{1}{2}\)?
   b) between \(\frac{1}{2}\) and 1?
   \[
   \frac{2}{5}, \frac{1}{4}, \frac{3}{8}, \frac{3}{8}, \frac{7}{12}, \frac{8}{10}, \frac{1}{3}, \frac{5}{6}
   \]
   How do you know?

4. Find each sum. What patterns do you see in the fractions and their sums?
   a) \(\frac{1}{2} + \frac{2}{1}\)
   b) \(\frac{2}{3} + \frac{3}{2}\)
   c) \(\frac{3}{4} + \frac{4}{3}\)
   d) \(\frac{4}{5} + \frac{5}{4}\)

5. Takoda and Wesley are collecting shells on the beach in identical pails. Takoda estimates she has filled \(\frac{7}{12}\) of her pail. Wesley estimates he has filled \(\frac{4}{10}\) of his pail.
   Suppose the children combine their shells. Will one pail be full? Explain.

6. Add.
   a) \(\frac{3}{4} + \frac{1}{8}\)
   b) \(\frac{3}{4} + \frac{3}{4}\)
   c) \(\frac{4}{10} + \frac{1}{8}\)
   d) \(\frac{2}{3} + \frac{1}{8}\)
   e) \(\frac{2}{3} + \frac{5}{9}\)
   f) \(\frac{1}{5} + \frac{2}{6}\)

7. Is this a magic square?
   How do you know?

   \[
   \begin{array}{ccc}
   \frac{3}{8} & \frac{1}{6} & \frac{11}{24} \\
   \frac{5}{12} & \frac{1}{3} & \frac{1}{4} \\
   \frac{5}{24} & \frac{1}{2} & \frac{7}{24}
   \end{array}
   \]

8. Subtract.
   a) \(\frac{3}{5} - \frac{1}{2}\)
   b) \(\frac{4}{3} - \frac{2}{7}\)
   c) \(\frac{3}{8} - \frac{3}{4}\)
   d) \(\frac{8}{5} - \frac{3}{2}\)

   a) \(3\frac{7}{10} - 2\frac{1}{5}\)
   b) \(4\frac{2}{3} - 1\frac{3}{8}\)
   c) \(2\frac{3}{4} - 1\frac{9}{10}\)
   d) \(4\frac{3}{8} - 3\frac{7}{10}\)

10. Farrah has run \(\frac{7}{10}\) of a race.
    Malcom has run \(\frac{6}{9}\) of the race.
    a) Who has run farther?
    b) How much farther?

11. Draw a rectangle on grid paper to find each product.
    a) \(\frac{7}{8} \times \frac{1}{2}\)
    b) \(\frac{1}{2} \times \frac{3}{4}\)
    c) \(\frac{3}{4} \times \frac{2}{3}\)
    d) \(\frac{2}{3} \times \frac{5}{4}\)

12. Multiply.
    a) \(\frac{4}{10} \times \frac{2}{3}\)
    b) \(\frac{7}{5} \times \frac{3}{8}\)
    c) \(2\frac{2}{3} \times 3\frac{3}{10}\)
    d) \(2\frac{2}{5} \times 2\frac{2}{5}\)

13. Aiko says that \(\frac{2}{3}\) of her stamp collection are Asian stamps.
    One-fifth of her Asian stamps are from India. What fraction of Aiko's stamp collection is from India?
When you first studied division, you learned two ways: sharing and grouping.
For example, \(20 \div 5\) can be thought of as:
- Sharing 20 items equally among 5 sets
- Grouping 20 items into sets of 5
Recall that multiplication and division are inverse operations.
We know: \(20 \div 5 = 4\)
So, we also know: \(4 \times 5 = 20\)

**Explore**

Work with a partner.
Suppose you have 5 cups of concentrate.
- A recipe for a bowl of punch calls for \(\frac{1}{4}\) cup of concentrate.
  How many bowls of punch can you make?
- A different recipe calls for \(\frac{3}{4}\) cup of concentrate for one bowl of punch.
  How many bowls of punch could you make if you used this recipe?
  Draw a diagram to illustrate your answers.

**Reflect & Share**

Compare your answers with those of another pair of classmates.
Did you solve the problems the same way?
If not, explain your method to your classmates.

**Connect**

Before we divide a whole number by a fraction, think about how we divide whole numbers.
- To find how many 3s are in 6, group 6 into 3s.
  We can show this on a number line.
  There are 2 groups of 3 in 6.

\[
\frac{6}{3} = 2
\]
To find how many thirds are in 6, divide 6 into thirds.

There are 18 thirds in 6.
Write this as a division statement.

\[ 6 \div \frac{1}{3} = 18 \]
Notice: \[ 6 \times 3 = 18 \]

Use the same number line to find how many two-thirds are in 6.

Arrange 18 thirds into groups of two-thirds.
There are 9 groups of two-thirds.
We write: \[ 6 \div \frac{2}{3} = 9 \]

Use the number line again to find how many five-thirds are in 6; that is, \[ 6 \div \frac{5}{3} \].

Arrange 18 thirds into groups of five-thirds.

There are 3 groups of five-thirds.
There are 3 thirds left over.
Think: What fraction of \( \frac{5}{3} \) is \( \frac{3}{3} \)?

From the number line, \( \frac{3}{3} \) is \( \frac{3}{5} \) of \( \frac{5}{3} \).
So, \[ 6 \div \frac{5}{3} = 3\frac{3}{5} \]

We can also use a number line to divide a fraction by a whole number. This is illustrated in the Example that follows.
Example

Divide.

a) \( \frac{1}{5} \div 4 \)  

b) \( \frac{3}{5} \div 4 \)

Solution

a) To find \( \frac{1}{5} \div 4 \), mark \( \frac{1}{5} \) on a number line. Divide the interval 0 to \( \frac{1}{5} \) into 4 equal parts.

Each part is \( \frac{1}{20} \).
So, \( \frac{1}{5} \div 4 = \frac{1}{20} \)

b) To find \( \frac{3}{5} \div 4 \), mark \( \frac{3}{5} \) on a number line. Divide the interval 0 to \( \frac{3}{5} \) into 4 equal parts.

To do this, divide the fifths into twentieths.

Arrange the 12 twentieths into 4 equal groups.

There are \( \frac{3}{20} \) in each group.
So, \( \frac{3}{5} \div 4 = \frac{3}{20} \)

Practice

1. Use a number line to find each quotient.
   a) i) \( 2 \div \frac{1}{3} \)  
      ii) \( 2 \div \frac{2}{3} \)

   b) i) \( 3 \div \frac{1}{4} \)  
      ii) \( 3 \div \frac{2}{4} \)  
      iii) \( 3 \div \frac{3}{4} \)

   c) i) \( \frac{4}{8} \div 2 \)  
      ii) \( \frac{4}{8} \div 4 \)  
      iii) \( \frac{4}{8} \div 8 \)
2. Find each quotient. Use number lines to illustrate the answers.
   a) \(2 \div \frac{1}{2}\)      b) \(3 \div \frac{1}{3}\)      c) \(3 \div \frac{2}{3}\)
   d) \(4 \div \frac{1}{4}\)      e) \(4 \div \frac{3}{4}\)      f) \(4 \div \frac{3}{4}\)
3. Find each quotient. Use number lines to illustrate the answers.
   a) \(\frac{1}{2} \div 2\)      b) \(\frac{1}{3} \div 3\)      c) \(\frac{2}{3} \div 3\)
   d) \(\frac{1}{4} \div 4\)      e) \(\frac{2}{4} \div 4\)      f) \(\frac{3}{4} \div 4\)
4. Use a number line to find each quotient.
   a) \(\frac{4}{5} \div 3\)      b) \(2 \div \frac{3}{8}\)      c) \(\frac{1}{2} \div 5\)
   d) \(6 \div \frac{3}{4}\)      e) \(4 \div \frac{2}{3}\)      f) \(\frac{5}{8} \div 2\)
5. Ioana wants to spend \(\frac{3}{4}\) of an hour studying each subject. She has 3 h to study. How many subjects can she study?
6. Why is \(\frac{2}{3} \div 4\) not the same as \(4 \div \frac{2}{3}\)? Use number lines in your explanation.
7. **Assessment Focus** Copy these boxes.

\[
\square \div \square
\]

a) Write the digits 2, 4, and 6 in the boxes to find as many division statements as possible.
   b) Which statement in part a has the greatest quotient?
      The least quotient? How do you know?
      Show your work.
8. The numbers \(\frac{9}{2}\) and \(3\) share this property:
   their difference is equal to their quotient.
   That is, \(\frac{9}{2} - 3 = \frac{3}{2}\) and \(\frac{9}{2} \div 3 = \frac{3}{2}\)
   Find other pairs of numbers with this property.
   Describe any patterns you see.

**Reflect**

When you divide a whole number by a proper fraction, is the quotient greater than or less than the whole number? Include an example in your explanation.
You have used grouping to divide 4 by \( \frac{2}{3} \): \( 4 \div \frac{2}{3} \)
You have used sharing to divide \( \frac{2}{3} \) by 4: \( \frac{2}{3} \div 4 \)
You will now investigate dividing a fraction by a fraction: \( \frac{2}{3} \div \frac{1}{4} \)

**Explore**

Work with a partner.
Use this number line to find how many quarters are in \( \frac{2}{3} \);
that is, find \( \frac{2}{3} \div \frac{1}{4} \).

Look at the quotient.
Try to find a method to calculate the quotient
without using a number line.
Use a different division problem to check your method.

**Reflect & Share**

Compare your method with that of another pair of classmates.
Does your method work with their problem? Explain.
Does their method work with your problem? Explain.

**Connect**

Here are two ways to divide fractions.

➤ Use common denominators.
To divide: \( \frac{3}{5} \div \frac{1}{4} \)
Write each fraction with a common denominator.
Since 5 and 4 have no common factors,
their common denominator is \( 5 \times 4 = 20 \).

\[
\frac{3}{5} = \frac{12}{20} \quad \frac{1}{4} = \frac{5}{20}
\]
When the denominators are the same, divide the numerators.

\[
\frac{3}{5} \div \frac{1}{4} = \frac{12}{20} \div \frac{5}{20}
\]

This means: How many five-twentieths are in \(\frac{12}{20}\)?

From the number line, this is: \(12 \div 5 = 2\frac{2}{5}\)

So, \(\frac{3}{5} \div \frac{1}{4} = 2\frac{2}{5}\)

Use multiplication.

Recall that another way to divide by 4 is to multiply by \(\frac{1}{4}\).

\[
12 \div 4 = 3 \quad \text{and} \quad 12 \times \frac{1}{4} = 3
\]

Since 4 can be written as \(\frac{4}{1}\),

dividing by 4 is the same as dividing by \(\frac{4}{1}\).

So, we can write \(12 \div \frac{4}{1} = 3 \times \frac{1}{4} = 3\).

Similarly, another way to divide by \(\frac{1}{4}\) is to multiply by 4.

\[
3 \div \frac{1}{4} = 12 \quad \text{and} \quad 3 \times \frac{4}{1} = 12
\]

We can use the same pattern to divide two fractions.

The fraction \(\frac{1}{4}\) is the reciprocal of the fraction \(\frac{4}{1}\).

That is, \(\frac{3}{5} \div \frac{1}{4} = \frac{3}{5} \times \frac{4}{1} = \frac{12}{5}\).

Up until now, we have divided a fraction by a lesser fraction.

We can use the same methods when we divide a fraction by a greater fraction or when we divide mixed numbers.

**Example**

Divide.

a) \(\frac{3}{4} \div \frac{5}{6}\)

b) \(1\frac{7}{8} \div 1\frac{1}{4}\)

**Solution**

a) \(\frac{3}{4} \div \frac{5}{6}\)

Use multiplication.

Dividing by \(\frac{5}{6}\) is the same as multiplying by \(\frac{6}{5}\).

\[
\frac{3}{4} \div \frac{5}{6} \quad \text{can be written as}
\]

\[
\frac{3}{4} \times \frac{6}{5} = \frac{3 \times 6}{4 \times 5} = \frac{3 \times \frac{3}{2 \times 5}}{\frac{10}{10}} = \frac{9}{10}
\]
b) \(1\frac{7}{8} \div 1\frac{1}{4}\)

Change the mixed numbers to improper fractions.

\[
1\frac{7}{8} = \frac{8}{8} + \frac{7}{8} \quad 1\frac{1}{4} = \frac{4}{4} + \frac{1}{4} \\
= \frac{15}{8} \quad = \frac{5}{4}
\]

So, \(1\frac{7}{8} \div 1\frac{1}{4} = \frac{15}{8} \div \frac{5}{4}\)

Use common denominators.

Since 4 is a factor of 8, the lowest common denominator is 8.

Multiply the numerator and denominator by 2: \(\frac{5}{4} = \frac{10}{8}\)

\[
\frac{15}{8} \div \frac{5}{4} = \frac{15}{8} \div \frac{10}{8} \quad \text{Since the denominators are the same,}
\]

\[
= \frac{15}{10} \quad \text{divide the numerators.}
\]

\[
= \frac{15 \div 5}{10 \div 5}
\]

\[
= \frac{3}{2}, \text{or } 1\frac{1}{2}
\]

**Practice**

1. Use a copy of each number line to illustrate each quotient.
   a) \(\frac{5}{6} \div \frac{1}{3}\)

   ![Number line](image)

   b) \(\frac{3}{4} \div \frac{1}{3}\)

   ![Number line](image)

2. Use multiplication to find each quotient.
   a) \(\frac{8}{5} \div \frac{3}{4}\)  
   b) \(\frac{9}{10} \div \frac{5}{3}\) 
   c) \(\frac{7}{2} \div \frac{4}{3}\)  
   d) \(\frac{1}{2} \div \frac{7}{6}\)

3. Use common denominators to find each quotient.
   a) \(\frac{7}{12} \div \frac{1}{4}\)  
   b) \(\frac{3}{5} \div \frac{11}{10}\) 
   c) \(\frac{5}{2} \div \frac{1}{3}\)  
   d) \(\frac{5}{6} \div \frac{9}{8}\)

4. Divide.
   a) \(1\frac{9}{10} \div 2\frac{2}{3}\)  
   b) \(2\frac{3}{4} \div 2\frac{1}{3}\) 
   c) \(3\frac{1}{2} \div 1\frac{4}{5}\)  
   d) \(1\frac{3}{8} \div 1\frac{3}{8}\)
5. Divide.
   a) \( \frac{5}{3} \div \frac{3}{5} \)  b) \( \frac{4}{9} \div \frac{4}{9} \)  c) \( \frac{1}{6} \div \frac{5}{2} \)  d) \( 1\frac{3}{4} \div 2\frac{9}{10} \)

6. a) Find each quotient.
   i) \( \frac{3}{4} \div \frac{5}{8} \)  ii) \( \frac{5}{8} + \frac{3}{4} \)  iii) \( \frac{7}{12} \div \frac{2}{5} \)
   iv) \( \frac{2}{5} \div \frac{7}{12} \)  v) \( \frac{5}{3} \div \frac{4}{5} \)  vi) \( \frac{4}{5} \div \frac{5}{3} \)

b) In part a, what patterns do you see in the division statements and their quotients?
   Write two more pairs of division statements that follow the same pattern.

7. **Assessment Focus**
   a) Copy the boxes below.
   Write the digits 2, 3, 4, and 5 in the boxes to make as many different division statements as you can.

   \[
   \underline{\text{\_\_\_\_\_}} \div \underline{\text{\_\_\_\_\_}}
   \]

   b) Which division statement in part a has the greatest quotient?
   The least quotient? How do you know?
   Show your work.

8. Which statement has the greatest value?
   Give reasons for your answer.
   a) \( 3\frac{1}{5} \times \frac{1}{2} \)  b) \( 3\frac{1}{5} \times \frac{2}{3} \)  c) \( 3\frac{1}{5} \div \frac{2}{3} \)
   d) \( 3\frac{1}{5} \div \frac{2}{1} \)  e) \( 3\frac{1}{5} + \frac{2}{3} \)  f) \( 3\frac{1}{5} + \frac{3}{2} \)

9. Write as many division questions as you can that have \( \frac{5}{6} \) as their quotient.

**Reflect**

When you divide two fractions, how can you tell, before you divide, if the quotient will be:
• greater than 1?
• less than 1?
• equal to 1?
Use examples in your explanation.
Recall that we can write a fraction as a division.
For example, $\frac{5}{2}$ can be written as $5 \div 2$.

**Explore**

Use the Glossary if you have forgotten what a unit fraction is.

Work with a partner.
You will need a calculator.

- Write all the unit fractions with denominators from 1 to 10.
  Write each fraction as a decimal.
  Use a calculator to check your answers.

- Choose 3 different proper fractions.
  Write each fraction as a decimal.
  Trade decimals with your partner.
  Order the decimals from least to greatest.

**Reflect & Share**

Compare your fractions and decimals with those of another pair of classmates.
Sort the decimals into two sets. Which attributes did you use?

**Connect**

Recall these two types of decimals.
- These are **terminating decimals**: 0.5, 0.76, 0.435
  Each decimal has a definite number of decimal places.
- These are **repeating decimals**:
  0.333...; 0.454 545...; 0.811 111...
  Some digits in each decimal repeat forever.

- To write a fraction as a decimal,
  divide the numerator by the denominator.
  For example, $\frac{4}{11} = 4 \div 11 = 0.363 \, 636 \, 36...$
  We write $\frac{4}{11} = 0.\overline{36}$, with a bar over the digits that repeat.
  When we use a calculator to divide, the calculator may round the last digit and display 0.363 636 364.
To write a terminating decimal as a fraction, look at these patterns.

\[
0.3 = \frac{3}{10}
\]

\[
0.03 = \frac{3}{100}
\]

\[
0.003 = \frac{3}{1000}
\]

\[
0.33 = \frac{33}{100}
\]

\[
0.0033 = \frac{33}{1000}
\]

The number of digits after the decimal point tells the power of 10 in the denominator:

0.333 is 333 thousandths.

\[
0.333 = \frac{333}{1000}
\]

10³ in the denominator
3 digits after the decimal point

0.4567 is 4567 ten-thousandths.

\[
0.4567 = \frac{4567}{10000}
\]

10⁴ in the denominator
4 digits after the decimal point

**Example 1**

Write each decimal as a fraction in simplest form.

a) \(0.365\)  

b) \(0.0054\)

**Solution**

a) \(0.365\)

There are 3 digits after the decimal point.

In fraction form, the denominator is \(10^3\), or 1000.

\[
0.365 = \frac{365}{1000}
\]

Write in simplest form.

5 is a common factor of 365 and 1000.

So, divide numerator and denominator by 5.

\[
\frac{365}{1000} = \frac{365 \div 5}{1000 \div 5} = \frac{73}{200}
\]

\[
0.365 = \frac{73}{200}
\]

b) \(0.0054\)

There are 4 digits after the decimal point.

In fraction form, the denominator is \(10^4\), or 10 000.

\[
0.0054 = \frac{54}{10000}
\]

2 is a common factor of 54 and 10 000.

\[
= \frac{27}{5000}
\]

Divide numerator and denominator by 2.
We use place value to order decimals.

Example 2

Order these decimals from least to greatest.
0.45, 0.45, 0.4, 0.45

Solution

All the decimals have 0 in the ones place and 4 in the tenths place. Compare digits in the hundredths place and beyond.

- 0.45 can be written 0.454 545...
- 0.45 can be written 0.455 555...
- 0.4 can be written 0.400 000
- 0.45 can be written 0.450 000

- 0.4 has 0 in the hundredths place. It is the least.
- 0.45 has 0 in the thousandths place.
- 0.45 has 4 in the thousandths place.
- 0.45 has 5 in the thousandths place. It is the greatest.

In order from least to greatest: 0.4, 0.45, 0.45, 0.45

Practice

1. a) Write each fraction as a decimal.
   i) \(\frac{2}{3}\)   ii) \(\frac{3}{4}\)   iii) \(\frac{4}{5}\)   iv) \(\frac{5}{6}\)   v) \(\frac{6}{7}\)

   b) How can you tell which fractions in part a repeat and which terminate?

2. Write each decimal as a fraction, in simplest form.
   a) 0.73   b) 0.765   c) 0.8765   d) 0.0006

3. For each fraction, write an equivalent fraction with a denominator that is a power of 10. Then, write the fraction as a decimal.
   a) \(\frac{1}{2}\)   b) \(\frac{2}{5}\)   c) \(\frac{3}{4}\)   d) \(\frac{13}{25}\)   e) \(\frac{19}{50}\)

4. Write each fraction as a decimal.
   a) \(\frac{2}{7}\)   b) \(\frac{3}{11}\)   c) \(\frac{2}{9}\)   d) \(\frac{5}{17}\)   e) \(\frac{5}{13}\)
5. In question 4d, the calculator display is not long enough to show the repeating digits. How could you find the repeating digits?

6. Write $\frac{1}{5}$ as a decimal.
   Use this decimal to write each number below as a decimal.
   a) $\frac{4}{5}$
   b) $\frac{7}{5}$
   c) $1\frac{4}{5}$
   d) $2\frac{1}{5}$

7. a) How many fractions can you write that are equivalent to the decimal 0.76?
   b) Have you written all possible fractions? Explain.

8. **Assessment Focus** Here is the Fibonacci sequence:
   1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...
   We can write consecutive terms as fractions:
   $\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{8}{5}$, $\frac{13}{8}$ and so on
   a) Write each fraction above as a decimal.
   What patterns do you see?
   b) Continue to write consecutive terms as decimals.
   Write about what you find out.

9. In each set, write the first three fractions as decimals.
   Look for a pattern.
   Use the pattern to write the remaining fractions as decimals.
   a) $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$
   b) $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, $\frac{7}{9}$
   c) $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$, $\frac{4}{11}$, $\frac{5}{11}$, $\frac{6}{11}$, $\frac{7}{11}$, $\frac{8}{11}$, $\frac{9}{11}$, $\frac{10}{11}$

10. Order the decimals in each set from least to greatest.
   a) 1.01, 0.1, 0.01, 0.1
   b) 1.3, 0.3, 2.3, 0.3, 0.35
   c) 0.46, 0.64, 1.46, 1.06, 0.6

**Reflect**

When you look at a decimal, how can you tell if it repeats or terminates?
Use examples in your explanation.
Recall that when you multiply a decimal by 10, the digits move 1 place to the left on a place-value chart, or the decimal point shifts 1 place to the right.

What happens when you multiply a decimal by 100? By 1000?

**Explore**

Work on your own.
Use a calculator.

➤ Choose a 4-digit decimal.
Divide it by 0.1, 0.01, and 0.001.
What patterns do you notice?

➤ Choose a different 4-digit decimal.
Use patterns to divide it by 0.1, 0.01, and 0.001.
Check your answers with a calculator.

**Reflect & Share**

Compare your strategies for dividing with those of a classmate.
How could you use multiplication to divide by 0.1, 0.01, and 0.001?

**Connect**

➤ Dividing by \(\frac{1}{10}\) is the same as multiplying by 10.

\[
0.1 = \frac{1}{10}
\]

So, \(1.35 \div 0.1 = 1.35 \div \frac{1}{10}\)

= \(1.35 \times 10\)

= 13.5

➤ Dividing by \(\frac{1}{100}\) is the same as multiplying by 100.

\[
0.01 = \frac{1}{100}
\]

So, \(1.35 \div 0.01 = 1.35 \div \frac{1}{100}\)

= \(1.35 \times 100\)

= 135
> Dividing by $\frac{1}{1000}$ is the same as multiplying by 1000.

So, $1.35 \div 0.001 = 1.35 \div \frac{1}{1000}$

$= 1.35 \times 1000$

$= 1350$

> Recall how to divide by powers of 10, such as 10 and 100.

$1.35 \div 10 = 0.135$

$1.35 \div 100 = 0.0135$

We can think of dividing by 10 as multiplying by $\frac{1}{10} = 0.1$.

So, $1.35 \div 10 = 1.35 \times 0.1$

$= 0.135$

And, dividing by 100 is the same as multiplying by $\frac{1}{100} = 0.01$.

So, $135 \div 100 = 135 \times 0.01$

$= 0.0135$

We can use these patterns to mentally divide by multiples of 0.1, 0.01, and 0.001.

**Example**

Divide.

a) $0.275 \div 0.2$

b) $1.863 \div 0.03$

**Solution**

a) $0.275 \div 0.2 = 0.275 \div \frac{2}{10}$

$= 0.275 \times \frac{10}{2}$

$= \frac{275}{2}$

$= 1.375$

b) $1.863 \div 0.03 = 1.863 \div \frac{3}{100}$

$= 1.863 \times \frac{100}{3}$

$= \frac{186.3}{3}$

$= 62.1$

**Practice**

Use mental math.

1. Predict the quotient when you divide each number by 100, 10, 1, 0.1, 0.01, and 0.001.

   a) 547   b) 879   c) 34.5   d) 6.52
   e) 6542.12   f) 0.234   g) 8.9   h) 10.01
2. Write each quotient.
   a) \( \frac{147}{1000} \)  b) \( \frac{147}{0.01} \)  c) \( \frac{9.64}{0.1} \)  d) \( \frac{12.30}{0.001} \)
   e) \( \frac{0.345}{0.01} \)  f) \( \frac{12.3}{10} \)  g) \( \frac{23.45}{0.01} \)  h) \( \frac{0.123}{0.001} \)

3. Find the missing divisor in each division statement.
   a) \( \frac{4.3}{?} = 4.3 \)  b) \( \frac{54}{?} = 5.4 \)  c) \( \frac{65.4}{?} = 6540 \)
   d) \( \frac{43.45}{?} = 434.5 \)  e) \( \frac{785.03}{?} = 7850.3 \)  f) \( \frac{0.0345}{?} = 3.45 \)
   g) \( \frac{0.00345}{?} = 0.345 \)  h) \( \frac{345.6}{?} = 3456 \)  i) \( \frac{0.593}{?} = 59.3 \)

4. Find the missing dividend in each division statement.
   a) \( \frac{?}{10} = 234 \)  b) \( \frac{?}{0.1} = 34.5 \)  c) \( \frac{?}{0.01} = 12.23 \)
   d) \( \frac{?}{0.001} = 12000 \)  e) \( \frac{?}{0.01} = 1320 \)  f) \( \frac{?}{0.001} = 50 \)
   g) \( \frac{?}{0.1} = 0.725 \)  h) \( \frac{?}{0.1} = 72.5 \)  i) \( \frac{?}{100} = 0.1456 \)

5. **Assessment Focus** A student says that when you divide two numbers, the quotient is always less than the dividend. Is this true? Use examples to explain your answer.

6. Find each quotient.
   a) \( 356.2 \div 0.2 \)  b) \( 127.5 \div 0.03 \)  c) \( 0.448 \div 0.4 \)
   d) \( 0.0525 \div 0.005 \)  e) \( 63.6 \div 0.06 \)  f) \( 211.4 \div 0.007 \)

7. A rectangle has an area of 15.5 \( \text{cm}^2 \).
   Find the length and perimeter of the rectangle for each width.
   a) 10 cm  b) 1 cm  c) 0.1 cm  d) 0.01 cm  e) 0.001 cm

8. a) Draw each rectangle on grid paper.
   i) 4 cm by 4 cm  ii) 6 cm by 4.4 cm  iii) 8.6 cm by 4.8 cm
   b) Calculate the area of each rectangle.
   c) What if you divide the area of the first rectangle by 0.1; the second by 0.01; the third by 0.001. Would each new rectangle be larger or smaller than the original rectangle? Explain.
   d) Find the area of each new rectangle in part c.
      Is each new rectangle similar to the original rectangle?
      Give reasons for your answer.

---

**Reflect**

Explain how you divide by 0.1, 0.01, and 0.001 mentally.
Use examples in your explanation.
When you designed your own math problem, you wrote a problem statement in the form of a question. You need to include the math information required to solve the problem. To know what information is needed, it is helpful to work backward from the problem statement.

Start with the problem statement.
For example, here is a problem from *Unit 2*:
How many desks are in the school?

To answer this, you might need to know this information:
- How many classrooms are in the school?
- How many desks are in each classroom?
- How many other desks are in the school?

You can investigate to find the information or you can make up your own information.
The problem might be written this way:
There are **12 classrooms**.
There are **30 desks in each classroom**.
There are **25 more desks in the library resource room**.
How many desks are in the school?

Some problems give more information than is needed.
It is helpful to **highlight the math information needed** to solve the problem and to **cross-out** information that is not needed.
Ensure that all information needed to solve the problem is provided. Keep in mind the strategies at the left that can be used to solve a problem.
1. a) In this problem, what extra information is needed to solve the problem? Shazi bought some 30¢ candy bars and some 60¢ candy bars. She bought 10 candy bars in total. How many candy bars did she buy at each price?

b) Make up the information you need to solve the problem in part a. Then solve the problem.

2. a) In this problem, what information is not needed to solve the problem? A bicycle dealer put together a shipment of bicycles and tricycles. Tricycles cost $25 more than bicycles. The dealer used 50 seats and 130 wheels. How many bicycles and how many tricycles did she put together?

b) Solve the problem in part a.

3. a) In this problem, what extra information is needed to solve the problem? The local hockey league has two divisions. Each division has 6 teams. How many games are played during the season?

b) Try to find out about a local hockey league in your area. Write a problem about the league. Solve the problem.

4. Write your own word problem.
Identify what math information is needed to solve it. Write the information needed to answer the problem, either by investigating to find the information or by making up your own information.
Solve the problem.
Trade problems with a classmate.
Solve your classmate's problem.
What Do I Need to Know?

- **To add or subtract two fractions:**
  Use equivalent fractions to make the denominators the same, then add or subtract the numerators. For example,
  \[
  \frac{5}{4} + \frac{3}{5} = \frac{23}{20} + \frac{12}{20} = \frac{37}{20}, \text{ or } 1\frac{7}{20}
  \]
  \[
  \frac{7}{3} - \frac{3}{8} = \frac{56}{24} - \frac{9}{24} = \frac{47}{24}, \text{ or } 1\frac{23}{24}
  \]

- **To multiply two fractions:**
  Multiply the numerators and multiply the denominators.
  \[
  \frac{2}{3} \times \frac{1}{5} = \frac{2\times 1}{3 \times 5} = \frac{2}{15}
  \]

- **To divide a whole number by a fraction:**
  Write the whole number as a fraction, then multiply.
  For \(4 \div \frac{2}{3}\), write: \(\frac{4}{1} \div \frac{2}{3}\) as \(\frac{4}{1} \times \frac{3}{2} = \frac{12}{2} = 6\)

- **To divide a fraction by a whole number:**
  Write the whole number as a fraction, then use common denominators.
  \[
  \frac{2}{3} \div 4 = \frac{2}{3} \div \frac{12}{3} = \frac{2}{12} = \frac{1}{6}
  \]

- **To divide two fractions:**
  **Method 1:**
  Use common denominators.
  \[
  \frac{4}{5} \div \frac{3}{2} = \frac{8}{10} \div \frac{15}{10}
  \]
  \[
  = \frac{8}{15}
  \]
  **Method 2:**
  Use multiplication.
  \[
  \frac{4}{5} \div \frac{3}{2} \text{ is the same as } \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}
  \]
LESSON

4.1 1. Name a fraction between each pair of fractions.
   a) \( \frac{1}{4} \) and \( \frac{1}{2} \)  
   b) \( \frac{1}{2} \) and \( \frac{3}{4} \)  
   c) \( \frac{1}{3} \) and \( \frac{3}{4} \)  
   d) \( \frac{3}{5} \) and \( \frac{7}{8} \)

2. Fletcher completed \( \frac{3}{5} \) of the test questions. Lalo completed \( \frac{2}{3} \) of the test questions.
   a) Who completed more questions?
   b) How many questions might have been on the test? Explain.

3. Order the fractions in each set from least to greatest.
   a) \( \frac{2}{3}, \frac{4}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{4} \)  
   b) \( \frac{7}{10}, \frac{1}{3}, \frac{3}{7}, \frac{3}{8}, \frac{2}{5} \)

4. Order the fractions in each set from greatest to least.
   a) \( \frac{1}{2}, \frac{3}{4}, \frac{7}{8} \)  
   b) \( \frac{4}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{12}, \frac{3}{12} \)  
   c) \( \frac{4}{5}, \frac{4}{6}, \frac{4}{10}, \frac{2}{3}, \frac{2}{4} \)

4.2 5. Add.
   a) \( \frac{3}{8} + \frac{3}{4} \)
   b) \( \frac{5}{6} + \frac{2}{7} \)
   c) \( \frac{3}{2} + \frac{5}{3} + \frac{9}{10} \)

4.3 6. Subtract.
   a) \( \frac{9}{10} - \frac{3}{4} \)
   b) \( \frac{19}{12} - \frac{1}{2} \)
   c) \( \frac{8}{9} - \frac{1}{8} \)
   d) \( \frac{7}{5} - \frac{7}{6} \)

4.4 7. Add or subtract as indicated.
   a) \( 2\frac{2}{3} + 1\frac{1}{2} \)
   b) \( 3\frac{1}{3} - 1\frac{7}{10} \)
   c) \( 2\frac{1}{6} + 4\frac{7}{8} \)
   d) \( 3\frac{1}{2} - 2\frac{3}{4} \)

8. A flask contains \( 2\frac{1}{2} \) cups of juice.
   Ping drinks \( \frac{3}{5} \) cup of juice.
   Preston drinks \( \frac{7}{10} \) cup of juice.
   How much juice is in the flask now?

9. Multiply. Use an area model to illustrate each product.
   a) \( \frac{2}{3} \times 15 \)
   b) \( \frac{7}{10} \times \frac{5}{8} \)
   c) \( 5 \times \frac{3}{2} \)
   d) \( \frac{2}{3} \times \frac{3}{8} \)
   e) \( \frac{4}{5} \times \frac{3}{10} \)
   f) \( \frac{9}{8} \times \frac{1}{5} \)
   g) \( \frac{10}{3} \times \frac{5}{2} \)
   h) \( \frac{11}{6} \times \frac{7}{4} \)

10. Twenty-five Grade 8 students are going on a school trip.
    They pre-order sandwiches.
    Three-quarters of the students order a turkey sandwich, while \( \frac{1}{4} \) of the students order a roasted vegetable sandwich. Of the \( \frac{3}{4} \) who want turkey, \( \frac{2}{5} \) do not want mayonnaise. What fraction of the students do not want mayonnaise?

11. Divide. Sketch a number line to show each quotient.
    a) \( 1 \div \frac{1}{3} \)
    b) \( 2 \div \frac{3}{4} \)
    c) \( 3 \div \frac{4}{5} \)
    d) \( 4 \div \frac{5}{6} \)

12. A glass holds \( \frac{2}{3} \) cup of milk.
    A jug holds 8 cups of milk.
    How many glasses can be filled from the milk in the jug?
13. Divide. Sketch a number line to show each quotient.
   a) \( \frac{3}{10} \div 2 \)  
   b) \( \frac{8}{5} \div 3 \)
   c) \( \frac{13}{2} \div 4 \)  
   d) \( \frac{5}{4} \div 3 \)

14. Jaiden estimates that he takes \( 1\frac{1}{4} \) h to knit a square for a blanket. How many squares can Jaiden knit in 25 h?

15. When you divide a fraction by a whole number, is the quotient greater than or less than 1? Include examples in your explanation.

   a) \( 6 \div \frac{2}{3} \)  
   b) \( \frac{3}{4} \div \frac{1}{4} \)
   c) \( \frac{1}{2} \div \frac{1}{4} \)  
   d) \( \frac{2}{3} \div \frac{3}{8} \)
   e) \( \frac{4}{5} \div \frac{3}{10} \)  
   f) \( \frac{9}{4} \div \frac{3}{2} \)
   g) \( \frac{12}{5} \div \frac{5}{12} \)  
   h) \( \frac{11}{7} \div \frac{11}{7} \)

17. Divide.
   a) \( \frac{5}{4} \div \frac{1}{3} \)  
   b) \( \frac{3}{8} \div \frac{9}{5} \)
   c) \( \frac{5}{2} \div \frac{5}{4} \)  
   d) \( \frac{7}{10} \div \frac{10}{3} \)

18. Divide.
   a) \( 1\frac{3}{4} \div 2\frac{1}{8} \)  
   b) \( 3\frac{5}{6} \div 2\frac{1}{5} \)
   c) \( 3\frac{1}{2} \div 1\frac{3}{8} \)  
   d) \( 2\frac{1}{5} \div 4\frac{2}{5} \)

19. When you divide a fraction by its reciprocal, is the quotient less than 1, greater than 1, or equal to 1? Use examples in your explanation.

20. Find each product and quotient.
   What patterns do you see?
   a) i) \( \frac{3}{1} \times \frac{1}{2} \)  
      ii) \( \frac{3}{1} \div 2 \)
   b) i) \( \frac{3}{4} \times \frac{2}{3} \)  
      ii) \( \frac{3}{4} \div \frac{3}{2} \)
   c) i) \( \frac{4}{5} \times \frac{3}{4} \)  
      ii) \( \frac{4}{5} \div \frac{4}{3} \)
   d) i) \( \frac{5}{6} \times \frac{2}{3} \)  
      ii) \( \frac{5}{6} \div \frac{3}{2} \)

   a) \( \frac{9}{8} - \frac{3}{4} \)
   b) \( \frac{9}{8} + \frac{3}{4} \)
   c) \( \frac{4}{3} \times \frac{5}{2} \)
   d) \( \frac{17}{10} \div \frac{2}{5} \)

22. Write each decimal as a fraction.
   a) 0.25  
   b) 0.75
   c) 0.32  
   d) 0.005

23. Write each fraction as a decimal.
   a) \( \frac{1}{8} \)
   b) \( \frac{3}{5} \)
   c) \( \frac{123}{250} \)
   d) \( \frac{19}{20} \)

24. Write each fraction as a decimal.
   a) \( \frac{2}{3} \)
   b) \( \frac{3}{7} \)
   c) \( \frac{3}{13} \)
   d) \( \frac{4}{11} \)

25. The tenths and hundredths digits of a decimal can be any digit from 0 to 9.
   a) Write all the decimals that are greater than \( \frac{1}{3} \) and less than \( \frac{3}{4} \).
   b) Order the decimals in part a from least to greatest.

26. Use mental math to divide.
   a) \( 57.8 \div 0.01 \)
   b) \( 0.882 \div 0.2 \)
   c) \( 1.374 \div 0.003 \)
1. Evaluate.
   a) \( \frac{7}{5} + \frac{3}{4} \)
   b) \( \frac{13}{10} - \frac{2}{3} \)
   c) \( \frac{3}{7} \times \frac{4}{9} \)
   d) \( \frac{5}{2} \div \frac{7}{6} \)

2. Which statement has the greatest value? How do you know?
   a) \( \frac{7}{3} \times \frac{3}{4} \)
   b) \( \frac{7}{3} - \frac{3}{4} \)
   c) \( \frac{7}{3} \div \frac{3}{4} \)
   d) \( \frac{7}{3} + \frac{3}{4} \)

3. Multiply a fraction by its reciprocal. What is the product?
   Use an example and an area model to explain.

4. a) Write \( \frac{1}{7} \) as a decimal.
   b) What is the 2001st digit in the repeating decimal for \( \frac{1}{7} \)?
      Explain how you know.

5. Which number is added to the numerator and denominator of \( \frac{2}{7} \)
   to get a fraction that is equivalent to \( \frac{1}{7} \)? Show your work.

6. Three-fifths of the Grade 8 class are in the band.
   a) On Tuesday, only \( \frac{1}{3} \) of these students went to band practice.
      What fraction of the class went to band practice on Tuesday?
   b) How many students might be in the class? How do you know?

7. Write each decimal as a fraction and each fraction as a decimal.
   a) \( \frac{7}{8} \)
   b) 0.64
   c) \( \frac{5}{11} \)
   d) 0.004

8. a) Choose a proper fraction.
    Add 1 to the numerator and to the denominator.
    Write the new fraction.
    Which fraction is greater?
    b) Repeat part a for 3 more different fractions.
       Is your answer about the greater fraction always the same?
       Explain.
Part 1

The side length of this square is 1 unit. Write the area of each of the 4 figures as a fraction of the area of the square.

Show how you used multiplication of fractions to find the areas. Order the fractions from least to greatest.

Part 2

What fraction of each square is shaded green?

Square A

Square B

Square C

Square D

How did you use the addition or subtraction of fractions to find each fraction?
Part 3

Draw a large square with side length 1 unit.
Draw line segments to divide the square into different figures.
Find the area of each figure as a fraction of the area of the square.
Copy the square without the fractions.
Trade squares with a classmate.
For your classmate’s square, write the area of each figure as a fraction of the area of the square.

Check List

Your work should show:
✓ how you used operations with fractions to solve the problems
✓ correct calculations and ordering of fractions
✓ an appropriate diagram of your problem
✓ clear explanations, with correct use of mathematical language

Reflect on the Unit

What do you know about fractions and decimals that you did not know before this unit? Use examples in your explanation.
1. The bird with the most feathers is the whistling swan.
   It has 25,216 feathers.
The ruby-throated hummingbird has the fewest feathers.
   It has 940 feathers.
   a) How many more feathers does the whistling swan have than the ruby-throated hummingbird?
   
   b) About how many hummingbirds together would have the same number of feathers as one whistling swan? Explain.

2. Write each number as a product of prime factors.
   Use exponents where possible.
   a) 38
   b) 15
   c) 252
   d) 105

3. According to Guinness World Records 2005, the greatest number of dominoes set up single-handed and toppled is 303,621 out of 303,628 by Ma Li Hua, in 2003.
   a) Write the number toppled in scientific notation.
   b) Write the number set up in scientific notation.
   c) What is the difference between the two numbers?
      Why can we not write this difference in scientific notation?

4. Copy each statement.
   Insert brackets to make each statement true.
   a) \[ 40 \div 5 + 3 \times 2^2 - 1 = 17 \]
   b) \[ 40 \div 5 + 3 \times 2^2 - 1 = 19 \]
   c) \[ 40 \div 5 + 3 \times 2^2 - 1 = 43 \]
   d) \[ 40 \div 5 + 3 \times 2^2 - 1 = 15 \]

5. Twelve less than a number is 13.
   Let \( x \) represent the number.
   Then an equation is \( x - 12 = 13 \).
   Solve the equation.
   What is the number?

6. A primary class is going to the zoo.
   The ratio of adults to children must be 2:7. Twenty-eight children go on the zoo trip. How many adults are needed for supervision?

7. The ostrich runs at 65 km/h.
   At this rate, how far can the ostrich run in 90 s?

8. There are 429 students registered at Woodside Public School.
   On Wednesday, about 0.7% of the students were absent.
   a) How many students were absent?
   b) What percent of students were at school?

9. A salesperson earns commission at a rate of 8%. Last week, she earned $450 commission. What were her total sales for the week?
10. Calculate the simple interest and the amount.
   a) $500 invested at an annual interest rate of 2% for 1 year
   b) $2750 invested at an annual interest rate of 3.5% for 4 years
   c) $4500 invested at an annual interest rate of 6.25% for 18 months

11. Use 9 linking cubes.
    Build an object.
    Draw as many views as a classmate would need to build the object.
    Trade views with a classmate.
    Build your classmate's object.
    Compare the object your classmate built from your views with the object you built.
    Explain any differences.

12. Here is the net of a triangular prism.

13. The base of a triangular prism has base $b$ and height $h$. The length of the prism is $l$.
    What are the possible values of $b$, $h$, and $l$ for a triangular prism:
    a) with volume 12 cm$^3$?
    b) with volume 24 cm$^3$?

14. Copy each pair of fractions.
    Use <, >, or = to compare the fractions.
    a) $\frac{2}{5}$ □ $\frac{6}{15}$
    b) $\frac{1}{9}$ □ $\frac{2}{18}$
    c) $\frac{8}{10}$ □ $\frac{3}{8}$
    d) $\frac{2}{3}$ □ $\frac{4}{5}$
    e) $\frac{3}{8}$ □ $\frac{2}{5}$
    f) $\frac{5}{6}$ □ $\frac{6}{7}$

15. Add or subtract.
    a) $\frac{2}{5} + \frac{1}{4}$
    b) $\frac{3}{8} + \frac{1}{2}$
    c) $\frac{7}{8} - \frac{1}{4}$
    d) $\frac{1}{2} - \frac{1}{10}$
    e) $\frac{7}{9} - 2\frac{1}{4}$
    f) $3\frac{1}{3} + 1\frac{1}{8}$

16. Which statement has the least value?
    a) $\frac{2}{3} + \frac{1}{6}$
    b) $\frac{2}{3} - \frac{1}{6}$
    c) $\frac{2}{3} \times \frac{1}{6}$
    d) $\frac{2}{3} \div \frac{1}{6}$
    e) $\frac{1}{6} \div \frac{2}{3}$
    f) $\frac{1}{6} \times \frac{2}{3}$

17. Write each fraction as a decimal.
    Then order the decimals from least to greatest.
    a) $\frac{13}{50}$
    b) $\frac{1}{4}$
    c) $\frac{51}{200}$
    d) $\frac{3}{11}$

18. Find each quotient.
    a) $\frac{3275}{0.1}$
    b) $\frac{3275}{0.01}$
    c) $\frac{3275}{0.001}$
    d) $\frac{3275}{0.5}$
    e) $\frac{3275}{0.05}$
    f) $\frac{3275}{0.005}$