In golf, the 2003 US Masters Championship was held in Georgia, USA. Mike Weir, a Canadian golfer, tied with Len Mattiace after 72 holes. Here are seven players, in alphabetical order, and their leader board entries:

- What was Weir's leader board entry?
- Order the entries from greatest to least.
- What other uses of integers do you know?

<table>
<thead>
<tr>
<th>Player</th>
<th>Over/Under Par</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim Furyk</td>
<td>-4</td>
</tr>
<tr>
<td>Retief Goosen</td>
<td>+1</td>
</tr>
<tr>
<td>Jeff Maggert</td>
<td>-2</td>
</tr>
<tr>
<td>Phil Mickelson</td>
<td>-5</td>
</tr>
<tr>
<td>Vijay Singh</td>
<td>-1</td>
</tr>
<tr>
<td>Mike Weir</td>
<td>-7</td>
</tr>
<tr>
<td>Tiger Woods</td>
<td>+2</td>
</tr>
</tbody>
</table>

Par for the tournament is 288.
Jim Furyk shot 284.
His score in relation to par is 4 under, or -4.

What You'll Learn

- Compare and order integers.
- Add, subtract, multiply, and divide integers.
- Use the order of operations with integers.
- Graph integers on a grid.
- Graph transformations on a grid.
- Solve problems involving integers.

Why It's Important

We use integers in everyday life, when we deal with weather, finances, sports, geography, and science.
Key Words

- positive integer
- negative integer
- opposite integers
- zero pair
- rational number
- coordinate grid
- quadrants
- ordered pair
- x-axis
- y-axis
- origin
Comparing and Ordering Integers

Positive integers, such as +5, +9, +1, are greater than 0.
Negative integers, such as −5, −9, −1, are less than 0.
Positive and negative integers can be shown on a number line:

Any positive integer is greater than any negative integer.
For example, +1 is greater than −1000.

We use the symbols > and < to show order.
−3 is to the left of +1; so, −3 is less than +1, and we write: −3 < +1
+3 is to the right of −4; so, +3 is greater than −4, and we write: +3 > −4

Example 1
Order these integers from least to greatest: +5, −6, +3, −8, 0, −1, +8

Solution
+5, −6, +3, −8, 0, −1, +8
Sketch a number line from −8 to +8. Mark a point on the line for each integer.

For least to greatest, read the integers from left to right: −8, −6, −1, 0, +3, +5, +8

Check

1. Sketch a number line. Use it to order these integers:
0, +2, −1, +4, −5, −7, +10

2. Copy each statement. Use < or > to show which number in each pair is greater. Use a number line if it helps.
a) +2 □ +8  b) 0 □ −5  c) −7 □ 0
d) +250 □ −251  e) −100 □ −70  f) −361 □ −360
Using Models to Add Integers

On a number line, opposite integers are the same distance from 0, but are on opposite sides of 0.

+3 and −3 are opposite integers.

We can use coloured tiles to model integers.

One red tile models −1. One yellow tile models +1.

+1 and −1 are opposite integers.

They combine to form a zero pair.

\((+1) + (-1) = 0\)

The sum of any two opposite integers is 0 because they form a zero pair:

\((+3) + (-3) = 0\) because +3 and −3 form 3 zero pairs.

We can use zero pairs to add integers with opposite signs.

Recall that when we add 0 to a number, the sum is equal to the number.

Example 2

Add: \((+5) + (-3)\)

Solution

Model +5 with 5 yellow tiles:
Model −3 with 3 red tiles:
Circle zero pairs.
There are 2 yellow tiles left. They represent +2.
So, \((+5) + (-3) = +2\)

To add integers with the same sign, we combine the tiles that represent the integers.

Then, we count the tiles.
Example 3

Add: \((-6) + (-4)\)

Solution

Model \(-6\) with 6 red tiles:

Model \(-4\) with 4 red tiles:

There are 10 red tiles altogether. They represent \(-10\).
So, \((-6) + (-4) = -10\)

✔ Check

3. Add.
   a) \((+3) + (+5)\)  
   b) \((-8) + (-11)\)  
   c) \((+6) + (+3)\)  
   d) \((-5) + (-6)\)  
   e) \((+5) + (+1)\)  
   f) \((-3) + (-6)\)  
   g) \((+4) + (-2)\)  
   h) \((-8) + (+5)\)  
   i) \((-5) + (+8)\)  
   j) \((-4) + (+2)\)  
   k) \((-9) + (+9)\)  
   l) \((-7) + (+2)\)

4. Write each scenario as a sum of two integers.
   Then, find the sum to answer each question.
   a) The temperature was \(-5^\circ\)C. It then rose \(8^\circ\)C.
      What was the final temperature?
   b) Keera earned $8 and spent $6. How much money did Keera have left?

Using Models to Subtract Integers

To subtract integers, we model the first integer with coloured tiles,
then remove the tiles that represent the second integer.
When we do not have enough tiles to remove, we can add zero pairs.

Example 4

Subtract: \((-3) - (-8)\)

Solution

\((-3) - (-8)\)
Model \(-3\) with 3 red tiles:
To take away $-8$, we need $5$ more red tiles.
Add $5$ zero pairs of tiles.
That is, add $5$ red tiles and $5$ yellow tiles. They represent $0$.

Take away $8$ red tiles.
$5$ yellow tiles remain. They represent $+5$.
So, $(-3) - (-8) = +5$

**Example 5**

Subtract: $(+2) - (-9)$

**Solution**

$(+2) - (-9)$
Model $+2$ with $2$ yellow tiles:
To take away $-9$, we need $9$ red tiles.
Add $9$ zero pairs of tiles.
That is, add $9$ red tiles and $9$ yellow tiles. They represent $0$.

Take away $9$ red tiles.
$11$ yellow tiles remain. They represent $+11$.
So, $(+2) - (-9) = +11$

**Check**

5. Subtract.
   a) $(+1) - (+5)$
   b) $(+4) - (+1)$
   c) $(+5) - (-9)$
   d) $(+8) - (-1)$
   e) $(+10) - (+5)$
   f) $(+10) - (-3)$

   a) $(+3) - (-8)$
   b) $(+7) - (-2)$
   c) $(-9) - (+3)$
   d) $(+5) - (-11)$
   e) $(+8) - (-8)$
   f) $(-5) - (+5)$
You have used coloured tiles to add integers. We will now investigate other ways to add.

**Explore**

Work with a partner.
Use these integers: $+5, -9, -16, +28, -34, +41$
Choose two integers. Add them.
Choose two different integers. Add them.
Repeat this activity.
Add as many different pairs of integers as you can.
Sketch a number line to show each sum.
Write each addition equation.

**Reflect & Share**

Compare your addition equations with those of another pair of classmates.
How can you add two integers when their signs are the same?
When their signs are different?

**Connect**

Recall how to use a number line to add integers.
The integer $+11$ is represented by an arrow, 11 units long, pointing right.
The integer $-5$ is represented by an arrow, 5 units long, pointing left.

To add: $(-5) + (+11)$
Start at $-5$ on a number line. Move 11 units right.

The arrow head is at $+6$.
So, $(-5) + (+11) = +6$
And, to add: \((-12) + (-3)\)
Start at \(-12\) on a number line and move 3 units left.

\(\begin{array}{c}
\end{array}\)

\((-12) + (-3) = -15\)

When you use a calculator to add, the keystrokes depend on the type of calculator.
When your calculator has this key \((-\)\), you use it to input the negative sign.
When your calculator has this key \(+/-\), you use it to change an input number to a negative number.

**Example 1**

Use a calculator to add. \((-325) + (-428)\)

**Solution**

\((-325) + (-428)\)

For a calculator with \((-)\):

Input:

\[
\begin{array}{c}
325 \, + \, (-) \, 428 \, = \\
\end{array}
\]

to display \(-753\)

So, \((-325) + (-428) = -753\)

**Example 2**

The temperature in Calgary, Alberta, was \(-2^\circ C\). A chinook came through and the temperature rose \(15^\circ C\). At nightfall, it fell \(7^\circ C\).

a) Write an addition expression to show the temperature changes.
b) Use a number line to find the final temperature.

**Solution**

a) The addition expression is: \((-2) + (+15) + (-7)\)
b) \((-2) + (+15) + (-7) = +6\)

The final temperature is \(+6^\circ C\).
1. Use a number line to add.
   a) \((+5) + (-6)\)    b) \((-8) + (+6)\)
   c) \((-2) + (-4)\)    d) \(0 + (-5)\)
   e) \((-2) + (-4)\)    f) \((-5) + (+5)\)

2. Use a number line to add.
   a) \((-3) + (+4) + (-6)\)    b) \((+3) + (-5) + (+7)\)
   c) \((+6) + (-8) + (-1)\)    d) \((-10) + (+6) + (-2)\)

3. a) Write the opposite integer.
    i) \(+8\)    ii) \(-5\)    iii) \(+2\)    iv) \(-8\)
   b) Add each integer to its opposite in part a.
   c) What do you notice about the sum of two opposite integers?

4. a) Add.
    i) \((+8) + (+6)\)    ii) \((+3) + (+7)\)
    iii) \((+5) + (+9)\)    iv) \((+1) + (+12)\)
   b) Look at the integer expressions and sums in part a.
   How are they related?
   How can you use this relationship to add two positive integers
   without a number line or a calculator?
   c) Check the relationship in part b by adding two positive integers
   of your choice.

5. a) Add.
    i) \((-8) + (-6)\)    ii) \((-3) + (-7)\)
    iii) \((-5) + (-9)\)    iv) \((-1) + (-12)\)
   b) Look at the integer expressions and sums in part a.
   How are they related?
   How can you use this relationship to add two negative integers
   without a number line or a calculator?
   c) Check the relationship in part b by adding two negative integers
   of your choice.

6. a) Find 4 pairs of integers that have the sum \(-5\).
   b) Find 4 pairs of integers that have the sum \(+4\).
7. **Assessment Focus** When you add two integers with opposite signs, the sum may be 0, a positive integer, or a negative integer. When you look at an addition expression, how can you tell which of these sums it will be, without adding? Include examples in your explanation.

8. Add.
   
   a) \((+513) + (-182)\)  
   b) \((+560) + (-266)\)  
   c) \((+793) + (-1089)\)  
   d) \((-563) + (+182) + (+363)\)  
   e) \((-412) + (+382) + (-79)\)  
   f) \((-114) + (+483) + (-293)\)

9. The value of a stock on the Toronto Stock Exchange has changed each week for six weeks as shown below:

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up $5</td>
<td>Down $6</td>
<td>Down $2</td>
<td>Up $4</td>
<td>Up $6</td>
<td>Down $2</td>
</tr>
</tbody>
</table>

   a) Write an integer addition expression to represent the change in the value of the stock at the end of Week 6.
   
   b) At the end of Week 6, how does the value of the stock compare with its value at the beginning of Week 1?
   
   c) At the beginning of Week 1, the stock was worth $40. How much was the stock worth at the end of Week 3? At the end of Week 6?

10. Find each missing integer.

   a) \(-5 = (-2) + \square\)  
   b) \(\square + (-8) = +2\)

11. Use only single-digit integers.

   How many ways can you complete each equation?

   How do you know you have found all possible ways?

   a) \(\square + \triangle = -2\)  
   b) \(\square + \triangle = -4\)

**Reflect**

Write an addition expression that has each sum:

a positive sum, a negative sum, and a sum of 0.

Show how to calculate each sum.
9.2 Subtracting Integers

Explore

Work with a partner.
You will need a calculator.
- Choose two positive integers between +150 and +300.
  Use a calculator to subtract them.
  Show the subtraction on a number line.
- Repeat the activity for a positive integer between +150 and +250, and a negative integer between −150 and −300.
- Repeat the activity for two negative integers between −150 and −300.

Reflect & Share

Compare expressions and number lines with those of another pair of classmates.
Suppose two integers are subtracted in reverse order.
What happens to their difference? Explain.
List the keystrokes to subtract two negative integers.

Connect

To subtract two integers, first think about how we subtract two whole numbers.
For example, to subtract: 13 − 6,
we think, “What do we add to 6 to get 13?”
The answer is 7, so: 13 − 6 = 7
To subtract two integers: (−6) − (+13),
we think, “What do we add to +13 to get −6?”

−19

We add −19; that is, (+13) + (−19) = −6
So, (−6) − (+13) = −19
We also know that: (−6) + (−13) = −19
This example shows that subtracting an integer is the same as adding the opposite integer.

\[ (-6) - (+13) = -19 \]
\[ (-6) + (-13) = -19 \]


**Example 1**

Use a number line to subtract.

a) \((+14) - (+30)\)  
b) \((-18) - (-12)\)

**Solution**

To subtract, add the opposite.

a) Write \((+14) - (+30)\) as \((+14) + (-30)\).

Use a number line.

Start at +14. Move 30 units left.

So, \((+14) - (+30) = -16\)

b) Write \((-18) - (-12)\) as \((-18) + (+12)\).

Use a number line.

Start at -18. Move 12 units right.

So, \((-18) - (-12) = -6\)

**Example 2**

The mean temperature in January for Victoria, BC, is \(+4{\degree}C\).

In Thunder Bay, the mean January temperature is \(-11{\degree}C\).

Which temperature is lower and by how much?

**Solution**

Since \(-11 < +4\), then \(-11{\degree}C\) is the lower temperature.

Subtract the temperatures to find their difference.

\[ (-11) - (+4) = (-11) + (-4) \]
\[ = -15 \]

The temperature in Thunder Bay is 15\(^\circ\)C lower than the temperature in Victoria.
Some integer expressions require addition and subtraction.

**Example 3**

Evaluate: \((+5) + (-3) - (+7)\)

**Solution**

\[
(+5) + (-3) - (+7) \quad \text{Write the subtraction as an addition.}
\]

\[
= (+5) + (-3) + (-7) \quad \text{Add the first two integers.}
\]

\[
= (+2) + (-7) \quad \text{Then add.}
\]

\[
= -5
\]

So, \((+5) + (-3) - (+7) = -5\)

We can use a calculator to subtract directly, without adding the opposite.

To use a calculator to subtract: \((-137) - (+542)\)

For a calculator with \([-\) : \(\text{For a calculator with } +/- :\)

Input: \([-] 137 \, [-] 542 \, =\) \(\text{Input: } 137 \, +/- \, [-] 542 \, =\)

to display \(-679\) \(\text{to display } -679\)

**Practice**

1. Rewrite as an addition statement. Then evaluate.
   a) \((+8) - (+4)\)  b) \((-13) - (-8)\)  c) \((-5) - (-5)\)
   d) \((+20) - (-16)\)  e) \((+30) - (-13)\)  f) \((+21) - (-18)\)

2. Use a number line to subtract.
   a) \((+7) - (+5)\)  b) \((+9) - (-3)\)
   c) \((-11) - (-4)\)  d) \((-14) - (+8)\)

   a) \((+4) - (+8)\)  b) \((-9) - (-5)\)  c) \((-7) - (+1)\)
   d) \((+10) - (-3)\)  e) \((+5) - (-5)\)  f) \((-18) - (-3)\)

4. Subtract.
   a) \((-256) - (+125)\)  b) \((-103) - (-214)\)
   c) \((+213) - (+133)\)  d) \((+148) - (-222)\)

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5. For each scenario:
   i) Write each number as an integer.
   ii) Subtract the second integer from the first integer.
       Explain each answer.
       a) A temperature 7°C above zero and
           a temperature 5°C below zero
       b) A temperature 15°C below zero and
           a temperature 8°C below zero
       c) A height 51 m above sea level and
           a depth 17 m below sea level
       d) A golf score of 2 over par and a golf score of 6 under par
       e) A rise of $21 in the value of a stock, then a fall of $14

6. The table shows the mean temperatures in January and July for several cities in a certain year.
   a) Find the difference between the temperatures in July and January for each city. Show your work.

<table>
<thead>
<tr>
<th>City</th>
<th>January Temperature (°C)</th>
<th>July Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Victoria</td>
<td>+6</td>
<td>+21</td>
</tr>
<tr>
<td>ii) Miami, US</td>
<td>+22</td>
<td>+31</td>
</tr>
<tr>
<td>iii) Winnipeg</td>
<td>−18</td>
<td>+20</td>
</tr>
<tr>
<td>iv) Perth, Australia</td>
<td>+25</td>
<td>+9</td>
</tr>
<tr>
<td>v) Calgary</td>
<td>−4</td>
<td>+25</td>
</tr>
</tbody>
</table>

   b) Which city has the greatest difference in temperatures? How do you know?
   c) In Perth, why is the July temperature less than the January temperature?

7. **Assessment Focus** For each integer below, write a subtraction expression that has this integer as its answer.
   a) −3
   b) +2
   c) 0

   Where possible, do this 4 different ways:
   • positive integer − positive integer
   • positive integer − negative integer
   • negative integer − positive integer
   • negative integer − negative integer

   Show your work.
8. Use the table below.

<table>
<thead>
<tr>
<th>City</th>
<th>Record High Temperature (°C)</th>
<th>Record Low Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Halifax, NS</td>
<td>+37</td>
<td>-29</td>
</tr>
<tr>
<td>Regina, SK</td>
<td>+43</td>
<td>-50</td>
</tr>
<tr>
<td>Thunder Bay, ON</td>
<td>+40</td>
<td>-41</td>
</tr>
<tr>
<td>Victoria, BC</td>
<td>+36</td>
<td>-16</td>
</tr>
</tbody>
</table>

a) Which city has the greatest record high temperature? The least record low temperature?
b) Find the difference between the temperatures for each city.
c) Which city has the greatest difference in temperatures?
d) What is the median record high temperature?
e) What is the range of record low temperatures?
f) Make up your own problem about these temperatures. Solve your problem.
Use a calculator to check your answers.

9. Evaluate.

a) \((-2) - (-8) - (+4)\)  
b) \((+5) - (-1) - (-3)\)

c) \((+10) - (+3) - (-7)\)  
d) \((-5) - (+8) - (+6)\)

e) \(0 + (-5) + (+8) + (-3)\)  
f) \((-42) + (-65) - (+28)\)

g) \((-1) - (+2) - (+3) - (+4)\)  
h) \((-241) - (+356) + (-5)\)

10. Write the next 3 terms in each pattern; then, write the pattern rule.

a) \(+5, +12, +19, \ldots\)  
b) \(-4, -2, 0, \ldots\)

c) \(-21, -17, -13, \ldots\)  
d) \(+1, 0, -1, \ldots\)

**Take It Further**

11. a) Find two integers with a sum of \(-12\) and a difference of \(+2\).

b) Create and solve a similar integer problem.

**Reflect**

When you subtract two integers, the answers can be positive, negative, or zero.

How can you predict the type of answer before you subtract?

Use examples in your explanation.
Adding and Subtracting Integers

The whole numbers are 0, 1, 2, 3, 4, ..., and so on.
The integers are ..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ..., and so on.
We do not need to include the + sign to indicate a positive integer.
So, all whole numbers are integers.

Up until now, when we added and subtracted integers, the integers were written in brackets.
For example, (−3) + (+7) and (−5) − (+8)

We will now interpret sums and differences of numbers such as −3 + 7 and −5 − 8.

Explore

Work with a partner.
Use a number line to find each answer.
(+3) + (−7) and 3 − 7
(−3) + (−7) and −3 − 7
(+3) + (+7) and 3 + 7
(−3) + (+7) and −3 + 7

What patterns do you see?
Write your own two sets of expressions like these, then evaluate.

Reflect & Share

Compare solutions with those of another pair of classmates.
How could you find each answer without a number line?

To add: −6 + 4, use a number line.

−6 + 4 = −2

We can also use mental math.
One number is negative: −6
One number is positive: +4
The answer to \(-6 + 4\) is the difference of 6 and 4, and the sign of the difference matches the sign of \(-6\), the numerically larger number:
\[-6 + 4 = -2\]

➢ To subtract: \(4 - 10\), use a number line.

\[-10\]
\[-7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4\]

\[4 - 10 = -6\]

Use mental math.
One number is positive: 4
One number is negative: \(-10\)
The answer to \(4 - 10\) is the difference of 10 and 4, and the sign of the difference matches the sign of \(-10\), the numerically larger number:
\[4 - 10 = -6\]

➢ To evaluate: \(-4 - 10\), use a number line.

\[-10\]
\[-15 \quad -14 \quad -13 \quad -12 \quad -11 \quad -10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3\]

\[-4 - 10 = -14\]

Use mental math.
Both numbers are negative.
The answer is the sum of 4 and 10, and the sign of the answer matches the sign of both numbers.
So, \(-4 - 10 = -14\)

### Example

Evaluate. \(8 - 6 - 9\)

### Solution

\[8 - 6 - 9\]

\[8 - 6 = 2\]

So, \(8 - 6 - 9 = 2 - 9\)

\[= -7\]
Practice

1. Evaluate.
   a) \(-5 + 7\)  b) \(3 + 4\)  c) \(-3 + 2\)
   d) \(-6 + 8\)  e) \(-11 + 13\)  f) \(-21 + 36\)
   g) \(5 - 12\)  h) \(-12 - 4\)  i) \(-6 - 9\)
   j) \(11 - 13\)  k) \(-5 - 18\)  l) \(15 - 3\)

2. Evaluate.
   a) \(6 - 1 + 3\)  b) \(-36 + 6 - 3\)  c) \(18 - 15 - 2\)

3. **Assessment Focus**
   a) Evaluate each pair of expressions.
      i) \(-6 + 4;\ 4 - 6\)  ii) \(-7 + 3;\ 3 - 7\)
      iii) \(-8 + 2;\ 2 - 8\)  iv) \(-9 + 1;\ 1 - 9\)
   b) What patterns do you see in the expressions and the answers?
      Explain why these patterns occur.
   c) Write two more expressions that are related in the same way as those in part a.
      Show your work.

4. Evaluate each term. Each pattern continues.
   Write the next 3 terms in each pattern.
   Explain how you found each answer.
   a) \((-6 + 5),\ (-7 + 4),\ (-8 + 3),\ ...\)
   b) \((-3 - 1),\ (-4 - 2),\ (-5 - 3),\ ...\)

5. Last Monday, Suneel had $283 in her bank account.
   On Tuesday, Suneel withdrew $120 in cash, and wrote a cheque for $200. On Thursday, Suneel deposited $53.
   How much money did Suneel have in her account then?
   Show your work.

Reflect

Do you prefer to use a number line or mental math to evaluate an expression involving integers?
Use an example in your explanation.
We can write the number of tiles in this array in two ways.
As a sum:
\[ 5 + 5 + 5 = 15 \]
As a product:
\[ (+3) \times (+5) = +15 \]

How can you use integers to write the number of tiles in this array in two ways?

In Explore, you will use patterns to find products such as \((-3) \times (+5)\) and \((-3) \times (-5)\), which cannot be represented as arrays.

---

**Explore**

Work on your own.
Your teacher will give you a large copy of this multiplication table.

Start at the bottom right of the table.
Multiply the positive integers.
Then complete the bottom left of the table.
Multiply a positive integer by a negative integer.
Use patterns or any method you wish to complete the top right, then the top left of the table.

---

**Reflect & Share**

Compare your completed table with that of a classmate.
Use the patterns in your table.
How could you multiply any negative integer by a positive integer?
How could you multiply any two negative integers?
These properties of whole numbers are also properties of integers.

**Multiplying by 0**

\[3 \times 0 = 0 \text{ and } 0 \times 3 = 0\]

So, \((-3) \times 0 = 0\) and \(0 \times (-3) = 0\)

**Multiplying by 1**

\[3 \times 1 = 3 \text{ and } 1 \times 3 = 3\]

So, \((-3) \times (+1) = -3\) and \((+1) \times (-3) = -3\)

**Order Property**

\[3 \times 4 = 12 \text{ and } 4 \times 3 = 12\]

So, \((-3) \times (+4) = -12\) and \((+4) \times (-3) = -12\)

**Distributive Property**

\[3 \times (4 + 5) = 3 \times 4 + 3 \times 5\]

\[= 12 + 15\]

\[= 27\]

So, \((+3) \times [(-4) + (-5)] = [(+3) \times (-4)] + [(+3) \times (-5)]\]

\[= (-12) + (-15)\]

\[= -27\]

From the introduction on page 380, you know that

\((+3) \times (+5) = +15\)

and \((+3) \times (-5) = -15\)

We can use the order property to show that:

Since \((+3) \times (-5) = -15\), then \((-5) \times (+3) = -15\)

We can use the distributive property to investigate the product of two negative integers.

Here are two ways to calculate: \((-5) \times [(+3) + (-3)]\)

**Method 1**

\[(-5) \times [(+3) + (-3)]\]

\[= (-5) \times (+3) + (-5) \times (-3)\]

\[= (-15) + (-5) \times (-3)\]

**Method 2**

\[(-5) \times [(+3) + (-3)]\]

\[= (-5) \times (+3) + (-5) \times (-3)\]

\[= (-5) \times (0)\]

\[= 0\]

These answers must be equal.
So, \((-15) + (-5) \times (-3) = 0\)
But: \((-15) + (+15) = 0\)
So, \((-5) \times (-3) = +15\)
We could do this with any pair of opposite integers in the square brackets.
So, the product of two negative integers is positive.

The results above can be used to write these rules for multiplying integers:
- The product of two integers with the same sign is positive.
  That is, \((+7) \times (+6) = +42\) and \((-7) \times (-6) = +42\)
- The product of two integers with opposite signs is negative.
  That is, \((+8) \times (-9) = -72\) and \((-8) \times (+9) = -72\)

To multiply more than two integers, we use the order of operations. That is, we multiply the integers in pairs, in the order they appear.

### Example

Find each product.

**a)** \((+3) \times (-6) \times (-2)\)

**b)** \((-2) \times (-4) \times (-5)\)

### Solution

**a)** \((+3) \times (-6) \times (-2)\)

Multiply the first two integers.

\[
\begin{align*}
&= (-18) \times (-2) \\
&= 36
\end{align*}
\]

**b)** \((-2) \times (-4) \times (-5)\)

Multiply the first two integers.

\[
\begin{align*}
&= (+8) \times (-5) \\
&= -40
\end{align*}
\]

When we write the product of integers, we do not need to write the multiplication sign.
That is, we may write \((-8) \times (-9)\) as \((-8)(-9)\).
And, \((+3) \times (-6) \times (-2)\) may be written as \((+3)(-6)(-2)\).
1. Will each product be positive or negative? How do you know?
   a) \((-6) \times (+2)\)  
   b) \((+6) \times (+4)\)  
   c) \((+4) \times (-2)\)  
   d) \((-7) \times (-3)\)  

2. Find each product.
   a) \((+8)(-3)\)  
   b) \((-5)(-4)\)  
   c) \((-3)(+9)\)  
   d) \((+7)(-6)\)  
   e) \((+10)(-3)\)  
   f) \((-7)(-6)\)  
   g) \((0)(-8)\)  
   h) \((+10)(-20)\)  
   i) \((-14)(-30)\)  

3. Find each product.
   a) \((-1)(-8)(-2)\)  
   b) \((-11)(-12)(-1)\)  
   c) \((-1)(-1)(-1)(-1)(-1)\)  
   d) \((-2)(-3)(-4)(-5)\)  

4. Copy each equation.
   Replace \(\square\) with an integer to make the equation true.
   a) \((+5) \times \square = +20\)  
   b) \(\square \times (-9) = +27\)  
   c) \((-9) \times \square = -54\)  
   d) \(\square \times (-3) = +18\)  
   e) \(\square \times (+5) = -20\)  
   f) \(\square \times (-12) = +144\)  
   g) \(\square \times (-6) = +180\)  
   h) \((+3) \times \square \times (-4) = +24\)  

5. Write the next 3 terms in each pattern.
   Then write the pattern rule.
   a) \(+1, +2, +4, +8, \ldots\)  
   b) \(+1, -6, +36, \ldots\)  
   c) \(-1, +3, -9, \ldots\)  
   d) \(-4, -8, -12, \ldots\)  

6. a) Find the product of each pair of integers.
   i) \((+3)(-7)\) and \((-7)(+3)\)  
   ii) \((+4)(+8)\) and \((+8)(+4)\)  
   iii) \((-5)(-9)\) and \((-9)(-5)\)  
   iv) \((-6)(+10)\) and \((+10)(-6)\)  

   b) Use the results of part a. Does the order in which integers are multiplied affect the product? Explain.

7. Use these integers: \(-5, +9, -8, +4, -2\)
   a) Which two integers have the greatest product?
   b) Which two integers have the least product?
   c) How do you know there is not a greater product or a lesser product?
**8. Assessment Focus**

a) Find each product. Then use a calculator to extend the pattern 4 more rows.
   
i) \((-2)(-3)\)   
   iii) \((-2)(-3)(-4)(-5)\)

b) Use the results of part a.
   
i) What is the sign of a product when it has an even number of negative factors? Explain.
   
   ii) What is the sign of a product when it has an odd number of negative factors? Explain.

   c) Investigate what happens when a product has positive and negative factors. Do the rules in part b still apply? Explain.

**9.** Explain why the product of an integer multiplied by itself can never be negative.

**Take It Further**

The natural numbers are 1, 2, 3, 4, ..., and so on.

**10.** How many different ways can you write \(-36\) as the product of two or more integer factors?

**11.** When you multiply two natural numbers, the product is never less than either of the two numbers. Is the same statement true for the product of any two integers? Investigate, then write what you find out.

**12.** The product of two integers is \(-144\).
The sum of the integers is \(-7\).
What are the two integers?

**13.** The product of two integers is between \(+160\) and \(+200\).
One integer is between \(-20\) and \(-40\).

   a) What is the greatest possible value for the other integer?
   
   b) What is the least possible value for the other integer?

**Reflect**

Suppose your friend missed this lesson. How would you explain to her how to multiply two integers? Use examples in your explanation.
Recall that, for any multiplication fact with two different factors, you can write two related division facts. For example:

\[ 9 \times 7 = 63 \]

So, \[ 63 \div 9 = 7 \] and, \[ 63 \div 7 = 9 \]

We can apply the same rules to the product of two integers.

**Explore**

Work with a partner.

Write each product below as many different ways as you can. For each product, write two related division facts.

- Write 75 as the product of two positive integers.
- Write 126 as the product of two negative integers.
- Write \(-72\) as the product of a negative integer and a positive integer.
- Write \(-80\) as the product of a positive integer and a negative integer.

**Reflect & Share**

Compare your division facts with those of another pair of classmates. Work together to develop rules for:

- dividing two positive integers
- dividing two negative integers
- dividing a negative integer by a positive integer
- dividing a positive integer by a negative integer

**Connect**

To divide integers, we use the fact that division is the inverse of multiplication.

We know that: \((+5) \times (+3) = +15\)

So, \((+15) \div (+5) = +3\) and \((+15) \div (+3) = +5\)

\[
\begin{array}{c}
\text{dividend} \\
\text{divisor} \\
\text{quotient}
\end{array}
\]

When the dividend and divisor are positive, the quotient is positive.
We know that: \((-5) \times (+3) = -15\)
So, \((-15) \div (+3) = -5\) and \((-15) \div (-5) = +3\)

When the dividend is negative and the divisor is positive, the quotient is negative.
When both the dividend and divisor are negative, the quotient is positive.

We know that: \((-5) \times (-3) = +15\)
So, \((+15) \div (-5) = -3\) and \((+15) \div (-3) = -5\)

When the dividend is positive and the divisor is negative, the quotient is negative.

The results above are true for all integers related in the different ways illustrated.

We can use these results to write rules for dividing integers:
- The quotient of two integers with the same sign is positive. That is, \((+56) \div (+8) = +7\) and \((-56) \div (-8) = +7\)
- The quotient of two integers with opposite signs is negative. That is, \((+63) \div (-9) = -7\) and \((-63) \div (+9) = -7\)

A division expression can be written with a division sign: \((-48) \div (-6)\); or as a fraction: \(\frac{-48}{-6}\)

When the expression is written as a fraction, we do not need to use brackets.

### Example

**Divide.**

a) \((-100) \div (-20)\)  
b) \(\frac{-30}{+5}\)

### Solution

a) Since the signs are the same, the quotient is positive.  
\((-100) \div (-20) = +5\)

b) Since the signs are different, the quotient is negative.  
\(\frac{-30}{+5} = -6\)

In *Example*, part b, the integer \(-6\) can be written as \(-\frac{6}{1}\). When the integer is written this way, it is written as a **rational number**.
1. Copy and continue each pattern until you have 8 rows.

Which rules for division of integers does each pattern illustrate?

\[
\begin{align*}
\text{a)} \quad (-12) \div (+3) &= -4 & \text{b)} \quad (+25) \div (-5) &= -5 \\
(-9) \div (+3) &= -3 & (+15) \div (-3) &= -5 \\
(-6) \div (+3) &= -2 & (+5) \div (-1) &= -5 \\
(-3) \div (+3) &= -1 & (-5) \div (+1) &= -5 \\
\text{c)} \quad (+8) \div (+2) &= +4 & \text{d)} \quad (+14) \div (+7) &= +2 \\
(+6) \div (+2) &= +3 & (+10) \div (+5) &= +2 \\
(+4) \div (+2) &= +2 & (+6) \div (+3) &= +2 \\
(+2) \div (+2) &= +1 & (+2) \div (+1) &= +2 \\
\text{e)} \quad (-14) \div (+7) &= -2 & \text{f)} \quad (-10) \div (-5) &= +2 \\
(-10) \div (+5) &= -2 & (-5) \div (-5) &= +1 \\
(-6) \div (+3) &= -2 & (0) \div (-5) &= 0 \\
(-2) \div (+1) &= -2 & (+5) \div (-5) &= -1
\end{align*}
\]

2. a) Use each multiplication fact to find a related quotient.

\[
\begin{align*}
\text{i)} & \quad \text{Given } (+8) \times (+3) = +24, \quad \text{find } (+24) \div (+3) = \square \\
\text{ii)} & \quad \text{Given } (-5) \times (-9) = +45, \quad \text{find } (+45) \div (-9) = \square \\
\text{iii)} & \quad \text{Given } (-7) \times (+4) = -28, \quad \text{find } (-28) \div (+4) = \square \\
\text{iv)} & \quad \text{Given } (+11) \times (-6) = -66, \quad \text{find } (-66) \div (+11) = \square \\
\end{align*}
\]

b) For each division fact in part a, write a related division fact.

3. Divide.

\[
\begin{align*}
\text{a)} \quad (+12) \div (-6) & \quad \text{b)} \quad (-9) \div (-3) & \quad \text{c)} \quad \frac{-20}{-5} \\
& \quad \frac{+21}{-7} & \quad (-32) \div (-8) & \quad (-144) \div (+12) \\
& \quad (-250) \div (+10) & \quad 0 \div (-8) & \quad (+125) \div (+5)
\end{align*}
\]

4. Nirmala borrowed $7 every day.
She now owes $56.
For how many days did Nirmala borrow money?

a) Write this problem as a division expression using integers.

b) Solve the problem.
5. Write the next three terms in each pattern. What is each pattern rule?
   a) \(-3, +9, -27, \ldots\)
   b) \(+6, -12, +18, -24, \ldots\)
   c) \(+5, +20, -10, -40, +20, +80, \ldots\)
   d) \(-64, +32, -16, \ldots\)
   e) \(+100000, -10000, +1000, \ldots\)

6. **Assessment Focus** Suppose you divide two integers. When is the quotient:
   a) less than both integers?
   b) greater than both integers?
   c) between the two integers?
   d) equal to \(+1\)?
   e) equal to \(-1\)?
   f) equal to \(0\)?
   Use examples to illustrate your answers. Show your work.

7. Divide.
   a) \((+624) ÷ (-52)\)
   b) \((-2231) ÷ (-23)\)
   c) \((-1344) ÷ (+16)\)
   d) \((-2068) ÷ (-47)\)

8. Evaluate.
   a) \((-32) ÷ (+4) ÷ (-2)\)
   b) \((-81) ÷ (-9) ÷ (-9)\)
   c) \((+56) ÷ (-4) ÷ (-2)\)

9. Find as many examples as you can of three different 1-digit numbers that are all divisible by \(+2\) and have a sum of \(+4\).

**Reflect**

How do you divide two integers? Include an example for each different possible division.
1. a) Write these integers in order from least to greatest: 
   +20, -4, -6, +13, 0, +2, -1
   b) Show these integers on a number line.

2. Use a number line to add.
   a) (-5) + (+7)
   b) (-10) + (+7)
   c) (-5) + (+12)
   d) (-3) + (+5) + (-4)

3. a) Add: (-18) + (+5)
   b) Find three other pairs of integers that have the same sum as part a.

4. Evaluate.
   a) (-7) - (+2)
   b) (-5) - (-2)
   c) (+4) - (-3)
   d) (-41) - (-17)
   e) (-3) - (+4) + (-5)

5. Evaluate.
   a) (-146) - (-571)
   b) (-365) + (-198) - (+118)

6. The price of a new car is $27 599. The value of the car decreases by $2600 each year for five years. What will the value of the car be after five years?
   a) Write an integer expression to represent this problem.
   b) Solve the problem.

7. Evaluate.
   a) 10 - 8 - 11
   b) -3 + 5 + 9
   c) -11 - 10 - 9
   d) 12 + 15 - 3

8. Multiply.
   a) (+8) \times (-4)
   b) (-120) \times (-10)
   c) (-4) \times (+7)
   d) (+6)(-12)
   e) (5)(0)(-1)
   f) (-4)(-8)(-1)

9. Use integers to answer each question. Show your work.
   a) The temperature drops 5°C, then drops another 3°C. What was the total drop in temperature?
   b) A swimming pool drains at a rate of 35 L/min for 30 min. How much water drained out of the pool?
   c) The price of a house rose $25 000, dropped $28 999, then rose $14 500. What was the total change in the price of the house?

10. Divide.
    a) (-81) \div (+9)
    b) (-12) \div (-6)
    c) 0 \div (-9)
    d) (+650) \div (-25)
    e) (-1288) \div (-28)
    f) (-100) \div (-100)
Recall the order of operations with whole numbers.
- Do the operations in brackets first.
- Do any work with exponents.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

The same order of operations applies to all integers.

**Explore**

Work with a partner.
Use these integers: \(-2, 4, -24, 7\)
Use any operations or brackets.
Write the expression that has the greatest value.

**Reflect & Share**

Share your expression with that of another pair of classmates.
If the expressions are different, check that the expression with the greater answer is correct.
Work together to write an expression that has the least value.

**Connect**

Since we use curved brackets to show an integer; for example, \((-2)\), we will use square brackets to group terms.

**Example 1**

Evaluate. \(100 - 3[20 ÷ (-2)]\)

**Solution**

\[
100 - 3[20 ÷ (-2)] \\
= 100 - 3 \times [20 ÷ (-2)] \quad \text{Do the operation in brackets first.}
\]

For clarity, we write positive integers as whole numbers.

\[
= 100 - 3(-10) \\
= 100 + 30 \quad \text{Multiply.}
\]

\[
= 130 \quad \text{Add.}
\]
When an expression is written as a fraction, the fraction bar has two meanings.
- The fraction bar indicates division.
- The fraction bar acts like brackets. That is, the operations in the numerator and denominator must be done before dividing the numerator by the denominator.

**Example 2**

Evaluate.

**a)** \[ \frac{4 \times (-8) + 2}{-6} \]

Multiply first.

\[
= \frac{-32 + 2}{-6}
\]

Add the integers in the numerator.

\[
= \frac{-30}{-6}
\]

Divide.

\[
= 5
\]

**b)** \[ 2 - (-3)^2 \]

Do the exponent first:

\[ (-3)^2 \] means \((-3)(-3)\)

Multiply.

\[ = 2 - (-3)(-3) \]

Subtract.

\[ = 2 - (+9) \]

\[ = -7 \]

**Practice**

1. a) Evaluate.
   i) \[ 12 \div (2 \times 3) - 2 \]
   ii) \[ 12 \div 2 \times (3 - 2) \]

b) Why are the answers different? Explain.

2. Evaluate. State which operation you do first.

a) \[ (+7)(+4) - (+5) \]

b) \[ (+6)[(+2) + (-5)] \]

c) \[ (-3) + (+4)(+7) \]

d) \[ (-6)(+4) \times (-2) \]

e) \[ (+15) \div [(+10) \div (-2)] \]

f) \[ (+18) \div (-6) \times (+2) \]

3. Evaluate.

a) \[ (-1)^3 \]

b) \[ (-3)^2 + 9 \]

c) \[ (5)^3 \times (-4) \]

d) \[ \frac{(-2)^3}{-4} \]

e) \[ \frac{(-6)(-8)}{4} \]

f) \[ \frac{(-12)(-3)}{-6} \]
4. Evaluate. Show all the steps.
   a) \((-3)(-2) + 4\)  
   b) \((-8)(-2) + (-1)\)  
   c) \(3(-4) - 2\)  
   d) \(-2(5 + 3)\)  
   e) \(10 \div 2 + 4 \times (-3)\)  
   f) \(\frac{(-7)(4) + 8}{(-2)^2}\)

5. **Assessment Focus** Robert, Brenna, and Christian got different answers for this problem: \(-40 - \frac{2}{(-8) \div 2}\)
   Robert’s answer was \(-32\), Christian’s answer was \(-48\), and Brenna’s answer was \(168\).
   a) Which student had the correct answer?
   b) Show and explain how the other two students got their answers. Where did they go wrong?

6. Which expression has a value that is closest to \(-500\)? Explain.
   \((-2)^2 \times (-100) \div 4 \times 5\)
   \(376 \div 4 \times (-5)\)
   \((-1360) \div 8 \times (-3)\)

7. Keisha had \$405\ in her bank account.
   Over the summer, she made 4 withdrawals of \$45\ each.
   What is the balance in Keisha’s account?
   Write an integer expression to represent this problem.
   Solve the problem.

8. The daily highest temperatures for one week in February were:
   \(-2^\circ C, +5^\circ C, -8^\circ C, -4^\circ C, -11^\circ C, -10^\circ C, -5^\circ C\)
   Find the mean temperature.

9. Write an expression for each statement.
   Evaluate each expression.
   a) Divide the sum of \(-24\) and \(4\) by \(-5\).
   b) Multiply the sum of \(-4\) and \(10\) by \(-2\).
   c) Subtract \(4\) from \(-10\), then divide by \(-2\).

**Take It Further**

**Reflect**

Make up an integer expression that has three operations.
Evaluate the expression. Show your work.
You have plotted points with whole-number coordinates on a grid.
Point A has coordinates (3, 2).
What are the coordinates of point B? Point C? Point D?

A vertical and a horizontal number line intersect at right angles at 0.
This produces a grid on which you can plot points with integer coordinates.

**Explore**

Work with a partner.
You will need grid paper and a ruler.
Copy this grid.

Draw a figure on the grid.
Make sure there is at least one vertex in each of the 4 parts on the grid.
Each vertex should be at a point where grid lines meet.
Label each vertex with a letter and the coordinates of the point.
List the vertices, with their coordinates, in order.
Trade lists with your partner.
Use the list to draw your partner’s figure.

**Reflect & Share**

Compare the figures you and your partner drew.
If they do not match, try to find which figure is incorrect, and why.
A vertical number line and a horizontal number line that intersect at right angles at 0 form a **coordinate grid**. The horizontal axis is the **x-axis**. The vertical axis is the **y-axis**. The axes meet at the **origin**. The axes divide the plane into four **quadrants**. They are numbered counterclockwise.

We do not need arrows on the axes.

A pair of coordinates is called an **ordered pair**.

We do not include a + sign for a positive coordinate.

In Quadrant 1, point A has coordinates (4, 6). In Quadrant 2, point B has coordinates (−4, 6). In Quadrant 3, point C has coordinates (−4, −6). In Quadrant 4, point D has coordinates (4, −6).

**History**

René Descartes lived in the 17th century. He developed the coordinate grid. It is named the Cartesian grid in his honour. There is a story that René was lying in bed and watching a fly on the ceiling. He invented coordinates as a way to describe the fly’s position.
Example

a) Write the coordinates of each point.
   i) P  ii) Q  iii) R  iv) S

![Graph showing points P, Q, R, and S on a coordinate grid.]

b) Plot each point on a grid.
   i) D(−1, 3)  ii) E(−3, −5)  iii) F(0, −2)  iv) G(−4, 0)

Solution

Remember, first move left or right, then up or down.

a) Start at the origin each time.
   i) To get to P, move 5 units right and 3 units down.
      So, the coordinates of P are (5, 3).
   ii) To get to Q, move 0 units right and 5 units down.
      So, the coordinates of Q are (0, −5).
   iii) To get to R, move 2 units left and 4 units down.
      So, the coordinates of R are (−2, −4).
   iv) To get to S, move 3 units left and 0 units down.
      So, the coordinates of S are (−3, 0).
b) i) D(−1, 3)
Start at −1 on the x-axis.
Move 3 units up. Mark point D.
ii) E(−3, −5)
Start at −3 on the x-axis.
Move 5 units down. Mark point E.
iii) F(0, −2)
Start at the origin.
Move 2 units down the y-axis. Mark point F.
iv) G(−4, 0)
Start at −4 on the x-axis.
Since there is no movement up or down,
point G lies on the x-axis. Mark point G.

Practice

1. Write the coordinates of each point from A to K.
2. Use the coordinate grid in question 1. Which points have:
   a) $x$-coordinate 0? 
   b) $y$-coordinate 0?
   c) the same $x$-coordinate?
   d) the same $y$-coordinate?
   e) equal $x$- and $y$-coordinates?
   f) $y$-coordinate  $+2$?

3. Draw a coordinate grid. Label the axes. Plot each point.
   a) A(6, $-6$) 
   b) B(5, 0) 
   c) C($-2$, 7)
   d) D($-3$, 8) 
   e) E(3, 1) 
   f) F(0, $-4$)
   g) O(0, 0) 
   h) H($-4$, $-1$) 
   i) J($-8$, 0)

4. Suppose you are given the coordinates of a point.
   You do not plot the point.
   How can you tell which quadrant the point will be in?

5. Draw a scalene triangle on a coordinate grid.
   Each vertex should be in a different quadrant.
   a) Label each vertex with its coordinates.
   b) What is the area of the triangle?

6. **Assessment Focus**
   Use a coordinate grid.
   How many different rectangles can you draw that
   have area 12 units$^2$?
   For each rectangle you draw, label its vertices.

7. a) Plot these points: K($-3$, 4), L(1, 4), M(1, $-2$)
   b) Find the coordinates of point N that forms rectangle KLMN.

8. a) Plot these points on a grid: A(5, $-7$), B($-3$, 3), and C(8, 8).
   Join the points.
   b) Find the area of $\triangle$ABC.

9. Plot the points C($-5$, 0) and D($-2$, $-3$).
   E is a point such that $\triangle$CDE is a right triangle.
   Find at least three possible positions for E.
   Write the coordinates of each point.

Choose four points, one in each quadrant. Write instructions to plot each point. Draw a grid to show your work.
Recall that a translation moves a figure in a straight line. When the figure is on a square grid, the translation is described by movements right or left, and up or down.

A translation and a reflection are transformations.

Which translation moved this figure to its image? A figure can also be reflected in a mirror line. Where is the mirror line that relates this figure and its image?

Explore

Work on your own.
You will need 0.5-cm grid paper and a ruler.
Draw axes on the grid paper to get 4 quadrants. Use the whole page.
Label the axes.
Draw and label a quadrilateral.
Each vertex should be where the grid lines meet.

➤ Translate the quadrilateral. Draw and label the translation image.
What do you notice about the figure and its image?

➤ Choose an axis.
Reflect the quadrilateral in this axis.
Draw and label the reflection image.
What do you notice about the figure and its image?

➤ Trade your work with that of a classmate.
Identify your classmate’s translation.
In which axis did your classmate reflect?

Reflect & Share

Did you correctly identify each transformation? Explain.
If not, work with your classmate to find the correct transformations.
To translate \( \triangle ABC \) 5 units right and 6 units down:
Begin at vertex \( A(-2, 5) \).
Move 5 units right and 6 units down to point \( A'(3, -1) \).
From vertex \( B(2, 3) \), move 5 units right and 6 units down to point \( B'(7, -3) \).
From vertex \( C(-5, 1) \), move 5 units right and 6 units down to point \( C'(0, -5) \).

Then, \( \triangle A'B'C' \) is the image of \( \triangle ABC \) after a translation 5 units right and 6 units down.
\( \triangle ABC \) and \( \triangle A'B'C' \) are congruent.

To reflect \( \triangle ABC \) in the \( y \)-axis:
Reflect each vertex in turn.
The reflection image of \( A(-2, 5) \) is \( A'(2, 5) \).
The reflection image of \( B(2, 3) \) is \( B'(-2, 3) \).
The reflection image of \( C(-5, 1) \) is \( C'(5, 1) \).

Then, \( \triangle A'B'C' \) is the image of \( \triangle ABC \) after a reflection in the \( y \)-axis.
\( \triangle ABC \) and \( \triangle A'B'C' \) are congruent.
The triangles have different orientations: we read \( \triangle ABC \) clockwise; we read \( \triangle A'B'C' \) counterclockwise.
Example

a) Plot these points: A(4, −4), B(6, 8), C(−3, 5), D(−6, −2)
Join the points to draw quadrilateral ABCD.
Reflect the quadrilateral in the x-axis.
Draw and label the reflection image A′B′C′D′.
b) What do you notice about the line segment joining each point to its reflection image?

Solution

a) After a reflection in the x-axis:
A(4, −4) → A′(4, 4)
B(6, 8) → B′(6, −8)
C(−3, 5) → C′(−3, −5)
D(−6, −2) → D′(−6, 2)

b) The line segments AA′, BB′, CC′, DD′ are vertical.
The x-axis is the perpendicular bisector of each line segment.
That is, the x-axis divides each line segment into 2 equal parts,
and the x-axis intersects each line segment at right angles.

In Practice question 6, you will investigate a similar reflection in the y-axis.

Practice

1. Identify each transformation. Explain your reasoning.
   a) 
   b) 

2. The diagram shows 4 parallelograms.

![Diagram of parallelograms]

a) Are any 2 parallelograms related by a translation? Explain.
b) Are any 2 parallelograms related by a reflection? Explain.

3. Copy this pentagon on grid paper.
   a) Draw the image after a translation 3 units left and 2 units up.
   b) Draw the image after a reflection in the x-axis.
   c) Draw the image after a reflection in the y-axis.

![Diagram of pentagon]

4. Plot these points on a coordinate grid:
   A(1, 3), B(3, -2), C(-2, 5), D(-1, -4), E(0, -3), F(-2, 0)
   a) Reflect each point in the x-axis.
      Write the coordinates of each point and its reflection image.
      What patterns do you see in the coordinates?
   b) Reflect each point in the y-axis.
      Write the coordinates of each point and its reflection image.
      What patterns do you see in the coordinates?
   c) How could you use the patterns in parts a and b to check that you have drawn the reflection image of a figure correctly?

5. a) Plot the points in question 4.
    Translate each point 4 units left and 2 units down.
   b) Write the coordinates of each point and its translation image.
      What patterns do you see in the coordinates?
   c) How could you use these patterns to write the coordinates of an image point after a translation, without plotting the points?
6. a) Plot these points on a coordinate grid:
P(1, 4), Q(−3, 4), R(−2, −3), S(5, −1)
Join the points to draw quadrilateral PQRS.
Reflect the quadrilateral in the y-axis.
b) What do you notice about the line segment joining each point to its image?

7. On a coordinate grid, draw a line through A(10, 10), O(0, 0), and B(−10, −10).
Use this line as the reflection line.
Draw a quadrilateral on one side of the line.
Draw its reflection image.
What patterns do you see in the coordinates of each point and its image?

8. a) Draw a figure and its image that could represent a translation and a reflection.
b) What attributes does the figure have?

9. **Assessment Focus** Draw a figure on a coordinate grid.
a) Choose a translation and/or a reflection that you could repeatedly apply to the figure and its images to make a design or pattern.
b) Does the figure tessellate? If your answer is no, could you change the transformation to make it tessellate? Explain.

10. You have transformed figures on a coordinate grid. Think about transformations in the real world.
a) Where do you see examples of translations?
b) Where do you see examples of reflections?

**Reflect**

How is a translation different from a reflection?
How are these transformations alike?
How can coordinate grids be used to illustrate these differences and similarities?
Graphing Rotations

Recall that a rotation turns a figure about the turn centre. The rotation may be clockwise or counterclockwise. The turn centre may be:

On the figure

Off the figure

How would you describe each rotation?

Explore

Work with a partner.
You will need 0.5-cm grid paper, tracing paper, a protractor, and a ruler.
Draw axes on grid paper to get 4 quadrants.
Place the origin at the centre of the paper.
Label the axes.
Draw and label a figure in the 1st quadrant.
Use the origin as the turn centre.
➢ Rotate the figure 90° counterclockwise.
    Draw its image.
➢ Rotate the original figure 180° counterclockwise.
    Draw its image.
➢ Rotate the original figure 270° counterclockwise.
    Draw its image.

What do you notice about the figure and its 3 images?

Reflect & Share

Compare your work with that of another pair of classmates.
What strategies did you use to measure the rotation angle?
Would the images have been different if you had rotated clockwise instead of counterclockwise? Explain.
To rotate the figure at the left clockwise:

- Trace the figure and the axes.
- Label the positive $y$-axis on the tracing paper.
- Rotate the tracing paper clockwise about the origin until the positive $y$-axis coincides with the positive $x$-axis.
- With a sharp pencil, mark the vertices of the image.
- Join the vertices to draw the image after a $90^\circ$ clockwise rotation about the origin, below left.

- Place the tracing paper so the figure coincides with its image.
- Rotate the tracing paper clockwise about the origin until the positive $y$-axis coincides with the negative $y$-axis.
- Mark the vertices of the image.
- Join the vertices to draw the image of the original figure after a $180^\circ$ clockwise rotation about the origin, above right.

- Place the tracing paper so the figure coincides with its second image.
- Rotate the tracing paper clockwise about the origin until the positive $y$-axis coincides with the negative $x$-axis.
- Mark, then join, the vertices of the image.
- This is the image after a $270^\circ$ clockwise rotation about the origin.

All 4 quadrilaterals are congruent.
A point and all its images lie on a circle, centre the origin.
**Example**

A counterclockwise rotation is shown by a positive angle such as +90°, or 90°. A clockwise rotation is shown by a negative angle such as −90°.

**Solution**

a) Plot these points: B(−5, 6), C(−3, 4), D(−8, 2)
Join the points to draw ΔBCD.
Rotate ΔBCD 90° about the origin, O.
Draw and label the rotation image ΔB′C′D′.

b) Join C, D, C′, D′ to O.
What do you notice about these line segments?

A rotation of 90° is a counterclockwise rotation.

a) Use tracing paper to draw the image ΔB′C′D′.
Rotate the paper counterclockwise until the positive y-axis coincides with the negative x-axis.

b) From the diagram,
OC = OC′ and OD = OD′
∠COC′ = ∠DOD′ = 90°

The *Example* illustrates these properties of a rotation:

- A point and its image are the same distance from the rotation centre.
- The angle between the segments joining a point and its image to the rotation centre is equal to the rotation angle.

In the *Practice* questions, you will verify these properties for other angles of rotation.

**Practice**

1. Each grid shows a figure and its rotation image.
   Identify the angle of rotation and the rotation centre.

   a)
   ![Diagram a]

   b)
   ![Diagram b]
2. Identify each transformation. Explain how you know.

3. a) Copy \( \triangle DEF \) on grid paper.

b) Rotate \( \triangle DEF \) \(-90^\circ\) about the origin to its image \( \triangle D'E'F' \).

c) Rotate \( \triangle DEF \) \(+270^\circ\) about the origin to its image \( \triangle D''E''F'' \).

d) What do you notice about the images in parts b and c? Do you think you would get a similar result with any figure that you rotate \(-90^\circ\) and \(+270^\circ\)? Explain.

4. Plot each point on a coordinate grid:

   A(2, 5), B(−3, 4), C(4, −1)

   a) Rotate each point \(180^\circ\) about the origin O to get image points A', B', C'.

   b) Draw and measure:

      i) OA and OA'
      ii) OB and OB'
      iii) OC and OC'

      What do you notice?

   c) Measure each angle.

      i) \( \angle AOA' \)
      ii) \( \angle BOB' \)
      iii) \( \angle COC' \)

      What do you notice?

   d) What other rotation of A, B, and C would result in the image points A', B', C'? Explain.

5. Repeat question 4 for a rotation of \(-90^\circ\) about the origin.
6. **Assessment Focus**  Draw and label 6 points on a grid, one in each quadrant and one on each axis.
   a) Rotate each point $-90^\circ$ about the origin.
      Write the coordinates of each point and its image.
      What patterns do you see in the coordinates?
   b) Repeat part a for a rotation of $180^\circ$ about the origin.
   c) Repeat part a for a rotation of $-270^\circ$ about the origin.
   d) How could you use the patterns in parts a, b, and c to draw a rotation image without using tracing paper?

7. You have rotated figures on a grid.
   Think about rotations in the real world.
   Where do you see examples of rotations outside of math class?

8. Draw a quadrilateral in the 3rd quadrant.
   a) Rotate the quadrilateral $180^\circ$ about the origin.
   b) Reflect the quadrilateral in the $x$-axis.
      Then reflect the image in the $y$-axis.
   c) What do you notice about the image in part a and the second image in part b?
      Do you think you would get a similar result if you started:
      i) with a different figure?  ii) in a different quadrant?
      Investigate to find out.
      Write about what you discover.

9. Plot these points: C(2, 6), D(3, -3), E(5, -7)
   a) Reflect $\triangle CDE$ in the $x$-axis to its image $\triangle C'D'E'$.
      Rotate $\triangle C'D'E'$ $-90^\circ$ about the origin to its image $\triangle C''D''E''$.
   b) Rotate $\triangle CDE$ $-90^\circ$ about the origin to its image $\triangle PQR$.
      Reflect $\triangle PQR$ in the $x$-axis to its image $\triangle P'Q'R'$.
   c) Do the final images in parts a and b coincide? Explain.

---

**Take It Further**

**Reflect**

When you see a figure and its transformation image on a grid, how can you identify the transformation? Include examples in your explanation.
Creating a Study Sheet

A study sheet helps you to review important math ideas. Your study sheet may be different from a classmate’s study sheet, but all study sheets should include the most important information from a unit. Add to and review your study sheet several times during a unit.

Here are some things to include on a study sheet.

- **Key Words**
  Record *Key Words* from the unit.
  Use a definition, a picture or example, and a problem.
  If any word is difficult to remember, create a word card. This card contains something to help you remember the meaning of the word.
  Use *Connect* and the *Illustrated Glossary* to help.

- **Formulas**
  Note any formulas or procedures that you may need to remember.
  Look in *Connect* for these.

- **Main Ideas**
  Select important main ideas for each lesson.
  Use lesson titles and *Focus* to organize the topics for the study sheet.
  Use *Reflect* from your notebook to help remember the main learning from the lessons. Highlight and summarize key points on your study sheet.

- **Sample Questions**
  Select sample questions for each lesson, such as the *Assessment Focus*, to review.
  Try doing the questions again.

- **Journal Notes**
  If you keep a journal, review your notes, then highlight and summarize things that you should remember on your study sheet.

- **Review Questions**
  Select sample questions from the *Unit Review* to practise.
Here is a study sheet for
Unit 8 Square Roots and Pythagoras.

**Key Words**

<table>
<thead>
<tr>
<th>Square Numbers</th>
<th>Square Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 4, 9, 16</td>
<td>( \sqrt{1} = 1 )</td>
</tr>
<tr>
<td>25, 36, 49</td>
<td>( \sqrt{16} = 4 )</td>
</tr>
<tr>
<td>64, 81, 100</td>
<td>( \sqrt{36} = 6 )</td>
</tr>
<tr>
<td></td>
<td>( \sqrt{81} = 9 )</td>
</tr>
</tbody>
</table>

**Formulas**

\[ c^2 = a^2 + b^2 \]

Pythagorean Theorem: In a right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

**Main Idea**
The Pythagorean Theorem can be used to find the length of one side in a right triangle when two other sides are known.

**Sample Question**
The size of a TV set is described by the length of a diagonal of the screen. One TV is labelled as size 70 cm.
The screen is 40 cm high.
What is the width of the screen?
Draw a diagram to illustrate your answer.

**Journal Notes**
To use the Pythagorean Theorem in an isosceles triangle, I draw a perpendicular from the vertex of the angle that is different from the equal angles, to the opposite side. Then I have two right triangles.

**Review Questions**
I need to review question 10, page 356, on finding the surface area of a pentagonal prism.
What Do I Need to Know?

- **Adding Integers**
  Use a number line.

  \[
  (-4) + (+6) = +2
  \]

- **Subtracting Integers**
  Add the opposite.
  Write \((-5) - (+4)\) as \((-5) + (-4)\), then add.

  \[
  (-5) + (-4) = -9
  \]

- **Multiplying Integers**
  The product of two integers with the same sign is a positive integer.

  \[
  (+6) \times (+4) = +24; \quad (-18) \times (-3) = +54
  \]

  The product of two integers with different signs is a negative integer.

  \[
  (-8) \times (+5) = -40; \quad (+9) \times (-6) = -54
  \]

- **Dividing Integers**
  The quotient of two integers with the same sign is a positive integer.

  \[
  (+56) \div (+8) = \frac{+56}{+8} = +7; \quad (-24) \div (-6) = \frac{-24}{-6} = +4
  \]

  The quotient of two integers with different signs is a negative integer.

  \[
  (-30) \div (+6) = \frac{-30}{+6} = -5; \quad (+56) \div (-7) = \frac{+56}{-7} = -8
  \]

- **Graphing on a Coordinate Grid**
  The coordinates of each point are:

  A(3, 2), B(-3, 2), C(-3, -2), and D(3, -2)

- **Transformations on a Coordinate Grid**
  A point or figure can be:
  - translated
  - reflected in the x-axis or the y-axis
  - rotated about the origin
LESSON

1. a) List these integers from least to greatest: +8, –10, –3, +1, –7
   b) Mark each integer in part a on a number line.

2. Use a number line to add or subtract.
   a) (–8) + (+5)
   b) (+14) + (–8)
   c) (–5) – (+3)
   d) (–7) – (–2)
   e) (+4) – (–3) + (–5)
   f) (–3) + (–8) – (–7)
   g) (+6) – (–10) + (–2)
   h) (–9) – (–11) – (–6)

3. Here are the results of a golf game.
   Al: –3; Lana: +2; Kirima: 0; Earl: +1; Reg: –5; Jody: –4
   a) Who won the game?
   b) How many fewer strokes did the winner take than the person who came last?

4. At midnight in Winnipeg, the temperature was –23°C. During the next 24 h, the temperature rose 12°C, then dropped 8°C. What was the final temperature? Explain.

5. Evaluate.
   a) (+512) + (–173)
   b) (–879) – (–1092)
   c) (–243) + (+987)
   d) (+1591) – (–847)

6. Evaluate.
   a) 3 – 5
   b) –1 + 10
   c) –5 – 6
   d) 3 – 5 + 7
   e) –4 + 3
   f) –3 + 5 – 7

7. The temperature change in a chemistry experiment was –2°C every 30 min. The initial temperature was 6°C. What was the temperature after 4 h?

8. Multiply.
   a) (–7)(–5)
   b) (+10)(–6)
   c) (–3)(–9)(–1)
   d) (–2)(–2)(–2)
   e) (–7)(–8)(0)
   f) (–11)(+13)(–2)

9. Evaluate.
   a) 21 – 5 – 5 + 6 – 2
   b) 0 – 6 + 4 + 8 – 1 + 2 + 7

10. Answer true or false to each statement. Explain your answer.
   a) The sum of an integer and its opposite is always 0.
   b) When you subtract two positive integers, their difference is always a positive integer.
   c) The product of a positive integer and a negative integer is always positive.
   d) The product of an integer and its opposite is always 0.
3.5 11. Divide.
   a) \((-56) ÷ (-7)\)
   b) \((+40) ÷ (-5)\)
   c) \((-121) ÷ (+11)\)
   d) \(-\frac{36}{-4}\)
   e) \(\frac{72}{-4}\)
   f) \(-\frac{28}{+2}\)

3.5 12. Evaluate.
   a) \((-8) ÷ (-4) + 6 × (-3)\)
   b) \((-5) + (-12) ÷ (-3)\)
   c) \(18 + 3[10 ÷ (-5)]\)
   d) \([(-16) ÷ 8]^2 - 12\)
   e) \(\frac{4 × (-5) - 4}{6}\)
   f) \(\frac{(-3)^2 + 5}{(-2)^2 - (-3)}\)

13. Evaluate.
   a) \(4[(-3) + 16]\)
   b) \(3 - 2(10 ÷ 2)\)
   c) \(5 × (-2) - 2[4 ÷ (-2)]\)
   d) \((-3)(-2)(4) + 3(-5)\)
   e) \(\frac{3 × (-6) - 3}{-7}\)
   f) \(9 - 3[(-2)^3 + 4]\)

14. In a darts game, Suzanne and Corey each threw the darts 10 times.
    Corey had: three \((+2)\) scores; three \((-3)\) scores; and four \((+1)\) scores.
    Suzanne had four \((+2)\) scores; four \((-3)\) scores; and two \((+1)\) scores.

   a) What was each person's final score?
   b) Who won the game? Explain.

15. For each number below, find two integers for which that number is:
   i) the sum
   ii) the difference
   iii) the quotient
   iv) the product
   a) \(-8\)  b) \(-2\)
   c) \(-12\)  d) \(-3\)

16. a) On a coordinate grid, plot each point. Join the points in order.
    Then join D to A.
    A\((-2, -2)\)  B\((6, -2)\)
    C\((3, 7)\)   D\((-5, 7)\)
    b) Name the quadrant in which each point is located.
    c) Identify the figure.
    Find its area.

17. a) Plot these points on a coordinate grid:
    A\((-2, 3)\), B\((-4, 0)\),
    C\((-2, -3)\), D\((2, -3)\)
    Join the points to draw quadrilateral ABCD.
    b) Draw the image of quadrilateral ABCD after each transformation:
       i) a translation 7 units left and 8 units up
       ii) a reflection in the x-axis
       iii) a rotation of 90° counterclockwise about the origin
    c) How are the images alike? Different?
1. Use a number line to order these integers from least to greatest.
   \(+5, -3, 0, -11, -8, +7\)

2. Evaluate.
   a) \((-4) + (-8)\)  
   b) \(9 + (-17)\)
   c) \((-8) \times 6\)  
   d) \((-56) + (-61)\)
   e) \((-10) - (-3)\)  
   f) \((4)(-2)\)
   g) \((-2)^4\)  
   h) \((-36) \div 9\)
   i) \((-3) \times (-5) \times (-11)\)

3. Continue each pattern. Write the next 4 terms.
   Write the pattern rule.
   a) \(-4, 8, -16, 32, \ldots\)  
   b) \(-9, -2, -5, 2, -1, 6, \ldots\)

4. Evaluate.
   a) \((-20) \times (-5) + 16 \div (-8)\)
   b) \(\frac{14 - 10 \div 2}{-5}\)
   c) \((-3)^2 + 2 \times (-4)\)

5. A number is multiplied by \(-4\).
   Then 3 is subtracted from the product.
   The answer is 13.
   What is the number?

6. The temperature on Sunday was \(4^\circ\)C. The temperature dropped \(8^\circ\)C on Monday and dropped twice as much on Tuesday.
   What was the temperature on Tuesday?

7. a) On a coordinate grid, draw a triangle with area 12 square units.
    Place each vertex in a different quadrant.
   b) Write the coordinates of each vertex.
   c) Explain how you know the area is 12 square units.
   d) Translate the triangle 6 units right and 3 units down.
   e) Reflect the triangle in the y-axis.
   f) Rotate the triangle 90° clockwise about the origin.
Charity Golf Tournament

A Grade 8 class and a local bank are partners to sponsor a golf tournament to raise money for local charities. The bank provides these prizes:

1st place—$5000 to a charity of the player’s choice
2nd and 3rd places—$1000 to a charity of the player’s choice

Golf Terms
Recall that “par” is the number of strokes it should take for a player to reach the hole.
If par is 3 and you take 5 strokes, then your score in relation to par is +2, or 2 over.
If par is 3 and you take 2 strokes, then your score in relation to par is −1, or 1 under.

A bogey is 1 stroke more than par, or 1 over par.
A double bogey is 2 strokes more than par, or 2 over par.
A birdie is 1 stroke less than par, or 1 under par.
An eagle is 2 strokes less than par, or 2 under par.

Here are the top 6 golfers:
Chai Kim, Delancy, Hamid, Hanna, Kyle, and Weng Kwong

1. The golf course has 9 holes. Here is one person’s results:
par on 3 holes, a bogey on 2 holes, a birdie on 1 hole,
an eagle on 2 holes, and a double bogey on 1 hole
a) Write an integer expression to represent these results.
b) Evaluate the expression in part a to calculate the score in relation to par.

2. Chai Kim wrote his results in a table like this.

<table>
<thead>
<tr>
<th>Hole</th>
<th>Par</th>
<th>Under/Over Par</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>−1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>+2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>−1</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>−1</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>0</td>
<td>−1</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

414 UNIT 9: Integers
a) Copy and complete the table. Use the following information:
   Hole 1, 4, 6, 7, 9  Par
   Hole 2, 5, 8  Birdie
   Hole 3  Double bogey

b) What was Chai Kim's final score?
c) What was his final score in relation to par?

3. For each person below, make a table similar to the table in question 2.
   Use the information below. What is each golfer's final score?
   a) Kyle:
      Bogey holes 1, 3, 5, 9
      Birdie hole 6
      Par holes 2, 4, 7, 8
   b) Delaney:
      Bogey holes 3, 4, 6
      Birdie holes 1, 2, 7, 8, 9
      Eagle hole 5
   c) Hamid:
      Birdie every hole except hole 8
      Double bogey hole 8

4. a) Hannah had a score of -5 in relation to par.
   Weng Kwong had a score of +3 in relation to par.
   Use the information in questions 2 and 3.
   Rank the players in order from least to greatest score.
   b) Who won the tournament and the $5000 prize?
      What was the score in relation to par?
   c) Who won the $1000 prizes?
      What were the scores in relation to par?

5. Use a table similar to that in question 2.
   Complete the table with scores of your choice.
   Calculate the final score, and the final score in relation to par.

What did you find easy about working with integers? What was difficult for you?
Give examples to illustrate your answers.