UNIT 11

Probability

- A quiz has two questions. Each question provides two answers. You guess each answer. What is the probability that you guess both answers correctly?

- Jason’s golden retriever is about to have two puppies. Jason wants to sell the puppies. He gets more money for female puppies. What is the probability that both puppies are female?

How are these two problems alike?

What You’ll Learn

- Identify 0 to 1 as a range of probabilities.
- List possible outcomes of experiments.
- Identify possible and favourable outcomes.
- Calculate probabilities from tree diagrams and lists.
- Compare theoretical and experimental probability.
- Apply probability to sports, weather predictions, and political polling.

Why It’s Important

You need to be able to make sense of comments you read or hear in the media relating to odds, chance, and probability.
Key Words

- experimental probability
- relative frequency
- theoretical probability
- simulation
- odds
Skills You'll Need

Experimental Probability

The experimental probability of an event occurring is:

\[
\text{Number of times the event occurs} \div \text{Total number of trials}
\]

The experimental probability may be written as a fraction, decimal, or percent. Experimental probability is also called relative frequency.

Example 1

In baseball, a batting average is a relative frequency of the number of hits. The number of times a player goes to bat is the player’s “at bats.”

This table shows data for 3 girls from a baseball team.

<table>
<thead>
<tr>
<th>Name</th>
<th>At Bats</th>
<th>Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abba</td>
<td>29</td>
<td>11</td>
</tr>
<tr>
<td>Gina</td>
<td>35</td>
<td>17</td>
</tr>
<tr>
<td>Stacy</td>
<td>42</td>
<td>23</td>
</tr>
</tbody>
</table>

Calculate the batting average for each player.

Solution

To calculate each player’s batting average, divide the number of hits by the number of at bats.

\[
\text{Abba} = \frac{11}{29} \approx 0.379 \\
\text{Gina} = \frac{17}{35} \approx 0.486 \\
\text{Stacy} = \frac{23}{42} \approx 0.548
\]

A batting average is always written with 3 decimal places.

Check

1. a) A quality control inspector tested 235 CDs and found 8 defective. What is the relative frequency of finding a defective CD?

b) A rug cleaning service uses telemarketing to get business. For every 175 telephone calls made, on average, there will be about 28 new customers. What is the experimental probability of getting a new customer over the phone?
Theoretical Probability

When the outcomes of an experiment are equally likely, the theoretical probability of an event is: \[ \frac{\text{Number of outcomes favourable to that event}}{\text{Number of possible outcomes}} \]

Usually we simply refer to the probability. We can use theoretical probability to predict how many times a particular event will occur when an experiment is repeated many times.

Example 2

A number cube is labelled from 1 to 6.

a) The number cube is rolled. What is the probability of getting a number less than 4?  
b) The cube is rolled 40 times. Predict how many times a number less than 4 will show.

Solution

When a number cube is rolled, there are 6 possible outcomes: 1, 2, 3, 4, 5, 6. The outcomes are equally likely.

a) A number less than 4 is 1, 2, or 3. So, 3 outcomes are favourable to the event “a number less than 4.” The probability of rolling a 1 or 2 or 3 is: \[ \frac{3}{6} = \frac{1}{2} \]

b) The predicted number of times a number less than 4 will show is: \[ \frac{1}{2} \times 40 = 20 \]

Check

2. The pointer on this spinner is spun 80 times. Is each statement true or false? Justify your answer.
   The pointer will land:
   a) on blue exactly 20 times
   b) on red or green about 40 times
   c) on each colour an equal number of times
   d) on each colour approximately 20 times
When your favourite hockey team is playing, you may ask: Will the team win? You may try to predict the outcome of the game. By making a prediction, you are estimating the probability that the event will occur.

**Explore**

Work on your own.

Use one of these words to describe the probability of each event below: impossible, unlikely, likely, certain

A Get a tail when a coin is tossed.
B Roll a 3 or 5 on a number cube labelled 1 to 6.
C Not roll a 3 or 5 on a number cube labelled 1 to 6.
D The same team will win the Grey Cup three years in a row.
E The sun will set tomorrow.
F A card drawn from a standard deck is a diamond.
G A card drawn from a standard deck is not a diamond.
H Roll a 4 on a number cube labelled 1 to 6.
I January immediately follows June.
J You will listen to a CD today.

Calculate the probability of each event where you can.

If you cannot calculate a probability, estimate it. Explain your estimate.

**Reflect & Share**

Compare your results with those of a classmate.

Arrange the events in order from impossible to certain.

What is the probability of an impossible event?
What is the probability of a certain event?

**Connect**

On page 455, you reviewed the method to calculate the theoretical probability of an event. When all the outcomes are favourable to that event, then the fraction:

\[
\frac{\text{Number of outcomes favourable to the event}}{\text{Number of possible outcomes}}
\]

has numerator equal to denominator, and the probability is 1.
When no outcomes are favourable to that event, then the fraction:
\[
\frac{\text{Number of outcomes favourable to the event}}{\text{Number of possible outcomes}}
\]
has numerator equal to 0, and the probability is 0.

So, the probability of an event occurring can be marked on a scale from 0 to 1.
When an event is impossible, the probability that it will occur is 0, or 0%.
When an event is certain, the probability that it will occur is 1, or 100%.
All other probabilities lie between 0 and 1.

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**Example**

Twenty cans of soup were immersed in water. Their labels came off. The cans are identical.
There are: 2 cans of chicken soup; 3 cans of celery soup; 4 cans of vegetable soup; 5 cans of mushroom soup; and 6 cans of tomato soup.
One can is opened.

a) What is the probability of each event?
   i) The can contains celery soup.
   ii) The can contains fish.
   iii) The can contains celery soup or chicken soup.
   iv) The can contains soup.

b) State which event in part a is:
   i) certain
   ii) impossible

---

**Solution**

a) There are 20 cans, so there are 20 possible outcomes.
   i) Three cans contain celery soup.

The probability of opening a can of celery soup is:
\[
\frac{3}{20} = \frac{15}{100} = 0.15, \text{ or } 15\%
\]

ii) None of the cans contains fish.

The probability of opening a can of fish is: 0, or 0%
iii) Three cans contain celery soup and two contain chicken soup.
This is 5 cans in all.
The probability of opening a can of celery soup or chicken soup is:
\[ \frac{5}{20} = \frac{25}{100} = 0.25, \text{ or } 25\% \]
iv) Since all the cans contain soup, the probability of opening a can of soup is:
\[ \frac{20}{20} = 1, \text{ or } 100\% \]
b) i) The event that is certain to occur is opening a can that contains soup.
This event has the greatest probability, 1.
ii) The event that is impossible is opening a can that contains fish.
This event has zero probability, 0.

In the Example, there are 17 cans that do not contain celery soup.
So, the probability of not opening a can of celery soup is \( \frac{17}{20} \). Then,
the probability of not opening a can of celery soup

\[ \frac{3}{20} + \frac{17}{20} = 1, \text{ or } 100\% \]

The events “opening a can of celery soup” and “not opening a can of soup” are complementary events. The sum of their probabilities is 1.

This is true in general:
the probability of an event occurring + the probability of an event not occurring = 1, or 100%

Practice

1. Copy this number line. Place the letter of each event on page 459 on the number line at the number that best matches the probability of the event.

<table>
<thead>
<tr>
<th>Impossible</th>
<th>Certain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>
a) Without looking, Jodi picks a pink block from a bucket with 5 pink blocks, 7 blue blocks, and 8 red blocks.
b) December immediately follows November.
c) Roll a 1 or 2 on a number cube labelled 1 to 6.
d) It will be warm in January in Ontario.
e) The colour of an apple is blue.

2. A TV weather channel reports a 35% chance of rain today. What is the probability that it will not rain today?

3. An election poll predicts that, in one riding, 4 out of 7 voters will vote for a certain political party. There are 25 156 voters. If the prediction is true, how many voters will not vote for that political party?

4. A magazine subscription service uses telemarketing to get business. For every 400 telephone calls, on average, there will be 88 orders for which 80 are actually paid.
a) What is the probability that, on any given call, a telemarketer will receive an order?
b) What is the probability that a telemarketer will receive an order that will be paid for?

5. Give an example of an event with each probability. Explain your choice.
   a) 1  b) $\frac{1}{6}$  c) $\frac{1}{2}$  d) $\frac{3}{4}$  e) 0

6. Light bulbs from a production line are tested. Of the 150 bulbs tested, 7 were defective.
a) What is the probability of a bulb not being defective? Describe two different ways to calculate this.
b) What if 60 356 bulbs are produced in a week. How many may be defective?

7. To play a board game, Marty and Shane roll a number cube labelled 1 to 6. The person with the greater number goes first. Marty rolls a 4.
a) What is the probability Marty will go first?
b) What is the probability Marty will not go first?
c) What is the probability that both Marty and Shane have to roll the number cube again? Explain.
8. **Assessment Focus**  A regular tetrahedron has 4 faces. The faces of one tetrahedron are labelled 1 to 4. The faces of the other tetrahedron are labelled 2 to 5. The two tetrahedrons are rolled. The numbers on the faces that land are added.

   a) Copy and complete this table for the possible outcomes.
   b) List the possible outcomes for the sum.
   c) Calculate the probability of each outcome.
   d) Add the probabilities in part c.

     What do you notice?
     Explain your result.

   e) Which event has each probability?

     i) 25%  
     ii) 75%  
     iii) 1  
     iv) 0

9. In the card game, *In Between*, a deck of cards is shuffled. Two cards are placed face up. A third card is dealt to the player. If the third card falls in between the two cards that are face up, the player wins. What is the probability that the player will not win for each pair of cards that are face up?

   a) a 2 of hearts and a 7 of spades   
   b) a 5 of diamonds and a king of clubs
   c) an ace of clubs and a 4 of spades   
   d) a 10 of diamonds and a jack of clubs

**Mental Math**

Divide each number by 0.1, 0.01, and 0.001:

- 578.25
- 0.365
- 42.011
- 1.432

What patterns do you see in your answers?

**Reflect**

Can the probability of an event be less than 1? Greater than 1? Explain.
Explore

Work with a partner.
You will need a number cube labelled 1 to 6 and a coin.

➢ One of you tosses the coin and one rolls the cube.
   Record the results.
➢ Calculate the experimental probability of the event “a head and a 2 show” after each number of trials.
   • 10 trials
   • 20 trials
   • 50 trials
   • 100 trials
➢ List the possible outcomes of this experiment.
➢ What is the theoretical probability of the event “a head and a 2 show”?
➢ How do the experimental and theoretical probabilities compare?

Reflect & Share

Compare your outcomes and probabilities with those of another pair of classmates.
Did you use a tree diagram to list the outcomes?
If not, work together to make a tree diagram.
Combine your results to get 200 trials.
What is the experimental probability of a head and a 2?
How do the experimental and theoretical probabilities compare?

Connect

Recall that a tree diagram can be used to find the outcomes of an experiment when the outcomes are equally likely.
Carina and Paolo play the *Same/Different* game with the spinner at the left. A turn is two spins.
Player A scores 1 point if the colours are the same.
Player B scores 1 point if the colours are different.

a) Use a tree diagram to list the possible outcomes of this game.
b) Find the probability of getting the same colours.
c) Find the probability of getting different colours.
d) Is this a fair game? Explain.
e) If you think this game is fair, give your reasons.
   If the game is not fair, how could you change the rules to make it fair?

**Solution**

a) The first branch of the tree diagram lists the equally likely outcomes of the first spin: blue, green, pink
   The second branch lists the equally likely outcomes of the second spin: blue, green, pink
   For each outcome from the first spin, there are 3 possible outcomes for the second spin.
   Follow the paths from left to right.
   List all the possible outcomes.

   ![Tree Diagram](tree-diagram.png)

   **First Spin**  |  **Second Spin**  |  **Possible Outcomes**
   --- | --- | ---
   blue | blue | blue/blue
   | green | blue/green
   | pink | blue/pink
   green | blue | green/blue
   | green | green/green
   | pink | green/pink
   pink | blue | pink/blue
   | green | pink/green
   | pink | pink/pink

b) From the tree diagram, there are 9 possible outcomes.
   Three outcomes have: blue/blue, green/green, pink/pink
   The probability of the same colour is:
   \[
   \frac{3}{9} = \frac{1}{3} = 0.33, \text{ or about } 33%\]
c) Six outcomes have different colours: blue/green, blue/pink, green/blue, green/pink, pink/blue, pink/green
   The probability of different colours is:
   \[ \frac{6}{9} = \frac{2}{3} = 0.67, \text{ or about 67\%} \]

d) The game is not fair. The chances of scoring are not equal.
   Player A can score only 3 out of the possible 9 points, while Player B can score 6 out of the possible 9 points.

e) Here is one way to make the game fair: Player A should score 2 points when the colours are the same. When each player has 9 turns, the players have equal chances of winning.

Carina and Paolo played the Same/Different game 100 times. There were 41 same colours and 59 different colours.

The experimental probability of the same colours is:
\[ \frac{41}{100} = 0.41, \text{ or 41\%} \]
The experimental probability of different colours is:
\[ \frac{59}{100} = 0.59, \text{ or 59\%} \]
These probabilities are different from the theoretical probabilities in the solution of the Example.
The greater the number of times the game is played, the closer the theoretical and experimental probabilities may be.

**Practice**

1. Here is a spinner game called *Make Green*.
   To play the game, a player spins the pointer on each spinner. To win, a player must get blue on Spinner 1 and yellow on Spinner 2, because blue and yellow make green. Your teacher will give you blank spinners. Use an open paper clip as a pointer.
   a) Make these spinners. Play the game 10 times. Record your results. How many times did you make green?
   b) Combine your results with those of 9 classmates. How many times did make green occur in 100 tries? What is the experimental probability for make green?
c) Use a tree diagram to list the possible outcomes for make green.
d) What is the theoretical probability for make green?  
e) How do the probabilities in parts b and d compare?

2. A number cube is labelled 1 to 6.  
A coin is tossed and the cube is rolled.  
Use the tree diagram you made in Reflect & Share.  
Find the probability of each event.  
a) a head and a 4  
b) a number less than 3  
c) a tail and a prime number  
d) not a 5

3. **Assessment Focus** Tara designs the game *Mean Green Machine*. A regular tetrahedron has its 4 faces coloured red, pink, blue, and yellow.  
A spinner has the colours shown.

When the tetrahedron is rolled,  
the colour on its face down is recorded.  
A player can choose to:  
• roll the tetrahedron and spin the pointer, or  
• roll the tetrahedron twice, or  
• spin the pointer twice

To win, a player must make green by getting blue and yellow.  
With which strategy is the player most likely to win?  
Justify your answer.  
Show your work.

4. Two tigers are born at the zoo each year for 2 years.  
A tiger is male or female.  
a) List the possible outcomes for the births after 2 years.  
b) What is the probability of exactly 1 male tiger after 2 years?  
c) What is the probability of at least 2 females after 2 years?  
d) What is the probability of exactly 2 females after 2 years?
A deck of playing cards has 52 cards. There are 4 suits: hearts, spades, diamonds, and clubs. There are 13 cards in each suit.

5. An experiment is: rolling a number cube labelled 1 to 6 and picking a card at random from a deck of cards. The number on the cube and the suit of the card are recorded.
   a) Use a deck of cards and a number cube.
      Carry out this experiment 10 times. Record your results.
   b) Combine your results with those of 9 classmates.
   c) What is the experimental probability of each event?
      i) a heart and a 4
      ii) a heart or a diamond and a 2
      iii) a red card and an odd number
   d) Draw a tree diagram to list the possible outcomes.
   e) What is the theoretical probability of each event in part c?
   f) Compare the theoretical and experimental probabilities of the events in part c.
      What do you think might happen if you carried out this experiment 1000 times?

6. Neither Andrew nor David likes to set the table for dinner. They toss a coin to decide who will set the table. David always picks tails. What is the probability David will set the table 3 days in a row? Justify your answer.

7. Each letter of the word WIN is written on a piece of paper. The pieces of paper are placed in a can. You make three draws.
   You place the letters from left to right in the order drawn. The pieces of paper are not replaced after each draw.
   If you draw the letters that spell WIN, you win a prize.
   a) What is the probability of winning a prize?
   b) What if the pieces of paper are replaced after each draw.
      Is the probability of winning increased or decreased? Explain.
   c) Design a similar contest using the letters of CRUISE.
      Repeat parts a and b.

Reflect

How is a tree diagram useful when calculating probabilities? Use an example in your explanation.
1. A TV news channel reports the results of a political poll. According to the poll, 53% of Canadians will reelect the current political party. What is the probability that the current political party will not be reelected?

2. Sal tossed a thumbtack 80 times. It landed point up 27 times. What is the experimental probability that the tack will land on its side?

3. Each number below is the probability of an event. Which of these words best describe the probability? certain, unlikely, likely, impossible
   a) 0.8  b) 0  c) 0.2  d) 0.4

4. A dentist tested a new whitener for teeth. She found that, in 1050 tests, it worked 1015 times.
   a) What is the experimental probability that the whitener works? That it does not work?
   b) The whitener is used on 8500 people. About how many people can expect whiter teeth?

5. Fari was at bat 211 times and hit 97 times. Eric was at bat 234 times and hit 119 times. Which player has the better batting average? Explain.

6. Use a number cube labelled 1 to 6. Roll it twice. Record the results. Repeat this experiment until you have 10 results. Combine your results with those of 9 classmates. You now have 100 results.
   a) Draw a tree diagram to list the possible outcomes.
   b) What is the theoretical probability of each event?
      i) rolling a 4 and a 5
      ii) rolling two even numbers
      iii) rolling the same number twice
      iv) rolling a 3
      v) not rolling a 1 or a 6
   c) What is the experimental probability of each event in part b?
   d) Compare the two probabilities for each event. What do you notice?

7. Red and black combine to make brown.
   a) Sketch two different spinners with pointers that could be spun to land on the two colours that combine to make brown.
   b) Draw a tree diagram to show the possible outcomes of spinning the two pointers.
   c) Calculate the probability of making brown.
A real-life situation can be simulated by a probability experiment. For certain events, a simulation is more practical than gathering data.

**Explore**

Work with a partner.
You will need a number cube labelled 1 to 6.
A store has a "Scratch and Save" day.
The store gives each customer a card with 6 circles on it.
Under the circles there are two matching percents and four different percents.
You scratch two circles.

If you get the two matching percents, you have that discount on all items you buy in the store that day.
The percents are arranged randomly on the card.
What is the probability you will scratch two matching percents?

➤ How can you use the number cube to estimate the probability you will scratch two matching percents?
➤ Conduct the experiment as many times as you can.
   Record each result.
➤ What is the experimental probability you will scratch two matching percents?

**Reflect & Share**

Compare your results with those of another pair of classmates. Are the probabilities equal? Explain.
Work together to sketch a spinner you could use, instead of a number cube, to carry out this experiment.
When we use a simulation to estimate probability, the model we use must have the same number of outcomes as the real situation. We use a coin when there are 2 equally likely outcomes.

We use a number cube, labelled 1 to 6, when there are 6 equally likely outcomes.

We use a spinner divided into congruent sectors, where the number of sectors matches the number of equally likely outcomes.

Example

In a Grade 8 class, for a group of 4 students, what is the probability that 2 or more students will have birthdays the same month? Design a simulation to find out.

Solution

There are 12 months in a year.
So, the probability of being born in a particular month is \( \frac{1}{12} \).
We need a simulation that has 12 equally likely outcomes.
Use a number cube labelled 1 to 6, and a coin.
For each number on the cube, assign a head or a tail.
Then, let each month be represented by one of these pairs:
January H1, February T1,
March H2, April T2,
May H3, June T3,
July H4, August T4,
September H5, October T5,
November H6, December T6
Toss a coin and roll a number cube 4 times;
one for each student in the group.
Record if any month occurred two or more times.
Conduct the experiment 100 times.
An estimate of the probability is:
The number of times a month occurred two or more times \( \frac{100}{100} \)
1. Work with a partner.  
You will need a coin and a number cube labelled 1 to 6.  
Conduct the experiment in the *Example* 25 times.  
Combine your results with those of 3 other pairs of students.  
Estimate the probability that, in a group of 4 people,  
2 or more people have birthdays in the same month.  

2. What if you want to estimate the probability that, in a group of  
6 students, at least 3 students have birthdays in the same month.  
How could you change the experiment in the *Example* to do this?  

3. a) When a child is born, the child is either female or male.  
What could you use to simulate this?  
b) You want to estimate the probability there are  
exactly 3 girls in a family of 4 children.  
Describe a simulation you could use.  
c) Conduct the simulation in part b.  
What is the estimated probability?  
d) Use a tree diagram to calculate the probability  
of exactly 3 girls.  
e) How do your answers to parts c and d compare? Explain.  

4. What if you wanted to estimate or calculate the probability of  
exactly 1 boy in a family of 4 children.  
How could you use the results in question 3 to do this?  

5. **Assessment Focus**  
A multiple-choice test has 5 questions.  
Each question has 4 answers.  
For each question, a student randomly chooses an answer.  
a) Design a spinner you could use to estimate the probability of  
getting 1 question correct.  
b) Make this spinner. Conduct a simulation to estimate the  
probability of getting 3 correct answers out of the 5 questions.  
c) How many times did you conduct the simulation?  
What is your estimate for the probability?  
Show your work.
6. Moira is on the school baseball team.  
On average, Moira gets 1 hit every 3 times at bat.  
Moira goes up to bat 4 times during a game.  
a) How can this spinner be used to simulate  
Moira’s batting average?  
Justify your answer.

b) Conduct the simulation 30 times. Copy and complete this table. Record the frequency of each number of hits per game.

<table>
<thead>
<tr>
<th>Hits per Game</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) Estimate the probability that, in a game,  
Moira will get each number of hits.  
i) 0 hits  ii) 1 hit  iii) 2 hits  iv) 3 hits  v) 4 hits

7. The weather forecast for each of the next 6 days is a 50% chance of rain.  
Describe a simulation to estimate the probability that it will rain on 3 of those 6 days.  
Conduct the simulation. Explain the result.

8. A basketball player has a 70% shooting average.  
He is about to take 2 foul shots.  
There is very little time remaining in the game, and his team is behind by 1 point.  
a) Describe a simulation to represent the player’s shooting ability.  
b) Conduct 20 simulations of the player’s 2 foul shots.  
c) What is the experimental probability that the player’s team will win the game?

Reflect

When would you use a simulation to estimate a probability?  
Include an example in your explanation.
Explore

Work with a partner.
You will need a number cube labelled 1 to 6, and a deck of cards.

➢ Suppose you roll the number cube.
  What is the probability of getting a number greater than 2?
  What is the probability of getting a number less than 2?
  Which event is more likely?
  How many times as likely is it?
➢ Suppose you remove a card, at random, from the deck.
  What is the probability it is a heart?
  What is the probability it is not a heart?
  Which event is more likely?
  How many times as likely is it?

Reflect & Share

How are the two events in each experiment alike?
How are they different?
Think of another experiment you could conduct that would have two events related the same way as those in Explore.

Connect

In the media, you may hear or read about the likelihood of the Toronto Maple Leafs winning the Stanley Cup.
This likelihood may be referred to in terms of odds.
For example, if the Maple Leafs are likely to win, then the odds in favour of their winning could be 5 to 1.
If the Maple Leafs are likely to lose, then the odds against their winning could be 3 to 1.
In a bag, there are 12 counters.  
A counter is picked at random.  
The number of outcomes favourable to the counter being yellow is 4.  
The number of outcomes not favourable to the counter being yellow is 8.  
So, the odds in favour of the counter being yellow are 4 to 8.  
This is a ratio, so it can be written in simplest form.  
Divide each term by 4.  
The odds in favour of the counter being yellow are 1 to 2.  
The odds against the counter being yellow are 2 to 1.  

In general,  
\[
\text{odds in favour} = \frac{\text{number of favourable outcomes}}{\text{number of unfavourable outcomes}}
\]

\[
\text{odds against} = \frac{\text{number of unfavourable outcomes}}{\text{number of favourable outcomes}}
\]

Example

A card is drawn at random from a deck of cards.  
a) What are the odds in favour of drawing a face card?  
b) What are the odds against drawing a face card?

Solution

There are 3 face cards in each suit: Jack, Queen, King  
There are 4 suits.  
So, there are \(3 \times 4\), or 12 favourable outcomes.  
There are 52 cards in the deck.  
So, there are \(52 - 12\), or 40 unfavourable outcomes.  
a) The odds in favour of drawing a face card are 12 to 40.  
Divide the terms by their common factor 4.  
The odds in favour of drawing a face card are 3 to 10.  
b) The odds against drawing a face card are 10 to 3.

Practice

1. What are the odds in favour of each event?  
a) Getting a number greater than 1 when a number cube labelled 1 to 6 is rolled
b) Getting a 2 when a card is randomly picked from a deck of playing cards
c) Getting the sum 7 when two number cubes labelled 1 to 6 are rolled and the numbers are added

2. What are the odds against each event in question 1?

3. What are the odds against each event?
   a) Getting a number less than 3 when a number cube labelled 1 to 6 is rolled
   b) Getting a black card when a card is randomly picked from a deck of playing cards
   c) Getting the sum 5 when two number cubes labelled 1 to 6 are rolled and the numbers are added

4. What are the odds in favour of each event in question 3?

5. Sera has 5 toonies, 8 loonies, 7 dimes, and 3 quarters in her piggy bank. One coin falls out at random.
   a) What are the odds in favour of the coin being a dime?
   b) What are the odds against the coin being a quarter?

6. The probability that David will be first in the 50-m backstroke is 25%. What are the odds in favour of his being first?

7. A weather report for Eastern Ontario was for a 40% probability of snow the next day.
   What are the odds against it snowing the next day?

8. **Assessment Focus** Use any materials or items you wish.
   a) Design an experiment where the odds in favour of one event are 3 to 7.
   b) Describe the experiment.
      List all possible outcomes and the odds in favour of each one.
      Show your work.

**Reflect**

Explain how odds in favour and odds against are related.
Include an example in your explanation.
Extending a Problem

- Read each question 1 to 7 carefully.
- Solve the problem using numbers, materials, pictures, graphs, tables, equations, and/or words.
  Try to find a solution in two different ways.
- Communicate your different solutions.
- Explain the solutions to someone else.
  Try to use only words in your explanation.
  Point to the numbers and other math information you used while you explain.
- Make up a similar problem by changing something (context, math information, problem statement). Solve the new problem.

1. You earn $20 a week doing yard work for a neighbour. You can either be paid the $20, or you can pull two bills from a brown paper bag containing:
   - two $5 bills
   - two $10 bills
   - one $20 bill
   For example, you might pull out a $5 bill followed by a $10 bill, and earn only $15. Or you might pull out the $20 bill followed by a $10 bill, and earn $30.
   Which is the better way for you to be paid? Justify your answer.

2. Use squares to make the next similar figure in this pattern.
   a) How many squares would be needed to make the 10th figure? The 20th figure?
   b) Suppose the pattern had been made with congruent equilateral triangles or congruent parallelograms. Would the numbers of figures needed to make the pattern change? Explain.

3. Suppose you have an unlimited supply of three masses: 3 kg, 11 kg, and 17 kg
   How many different ways can you combine at least one of each mass to make a total mass of exactly 100 kg?
4. Myles’ car has an automatic transmission. He drove 8 km. He spent 2 min at a stoplight. Marlee’s car has a manual shift. She drove 9 km with no stops. Use the table below. Who used more gas?

<table>
<thead>
<tr>
<th>Action</th>
<th>Automatic</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idling</td>
<td>0.16 L/min</td>
<td>0.16 L/min</td>
</tr>
<tr>
<td>Starting</td>
<td>0.05 L</td>
<td>0.05 L</td>
</tr>
<tr>
<td>Driving</td>
<td>1 L/22 km</td>
<td>1 L/20 km</td>
</tr>
</tbody>
</table>

5. This staircase has 3 steps. How many blocks would you need for a 20-step staircase?

6. The cost of renting a truck for a day is $45. There is an additional charge of $0.20/km. The cost, \( C \) dollars, for renting the truck and driving it \( k \) kilometres is \( C = 45 + 0.2k \).
   a) The truck is driven 70 km. What was the rental cost?
   b) Armand rented the truck. He paid a total of $65. How far was the truck driven?
   c) Jasmine rented the truck. She paid a total of $58.20. How far did she drive the truck?

7. The area of Square F is 16 square units. The area of Square B and of Square H is 25 square units. The measure of each side is a whole number of units. What is the area of each square?
   a) A  b) C  c) D  d) E  e) G

8. This table shows the temperatures on 5 consecutive days in four Canadian cities during February. Create a math problem from these data. Solve the problem.

<table>
<thead>
<tr>
<th>City</th>
<th>City</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kamloops, BC</td>
<td>Fredericton, NB</td>
<td>-10°C</td>
<td>-6°C</td>
<td>-3°C</td>
<td>-5°C</td>
<td>-6°C</td>
</tr>
<tr>
<td>Windsor, ON</td>
<td>Swift Current, SK</td>
<td>3°C</td>
<td>2°C</td>
<td>-1°C</td>
<td>-4°C</td>
<td>0°C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-5°C</td>
<td>4°C</td>
<td>6°C</td>
<td>8°C</td>
<td>6°C</td>
</tr>
</tbody>
</table>
What Do I Need to Know?

☑ Experimental Probability
The experimental probability of an event is:
- The number of times the event occurred
- The number of times the experiment is conducted

Experimental probability is also called relative frequency.

☑ Theoretical Probability
The theoretical probability of an event is:
- The number of outcomes favourable to that event
- Total number of outcomes

☑ Probability Range
All probabilities are greater than or equal to 0, and less than or equal to 1.
The probability of an event that is impossible is 0, or 0%.
The probability of an event that is certain is 1, or 100%.

\[
\left( \text{The probability that an event occurs} \right) = 1 - \left( \text{The probability that the event does not occur} \right)
\]

The sum of the probabilities of all possible outcomes is 1.

☑ Odds
Odds in favour = number of favourable outcomes to number of unfavourable outcomes
Odds against = number of unfavourable outcomes to number of favourable outcomes

These events are complementary events.

Math Link
Your World
The Princess Margaret Hospital Foundation runs a lottery every year to raise money for the hospital. One year, the chances of winning were given as 1 in 15. So, the odds in favour of winning were 1 to 14.
What Should I Be Able to Do?

For extra practice, go to page 498.

**Lesson**

1. a) What is the probability of a certain event?
   b) What is the probability of an impossible event?
   c) Estimate or calculate the probability of each event.
      i) Chris randomly picks an orange out of a basket that contains 2 oranges, 6 apples, and 8 peaches.
      ii) March immediately follows April.
      iii) Roll a 1, 2, 3, or 4 on a number cube labelled 1 to 6.
      iv) It will be cold in January in the Arctic.
      v) You will have homework tonight.

2. A TV weather channel reports there is a 60% chance of snow tomorrow.
   What is the probability that it will not snow tomorrow?

3. An election poll predicts that, in a certain riding, 5 out of 8 voters will vote for the Liberal Party in the next provincial election. According to this prediction, how many voters out of 180 000 voters would not vote Liberal?

4. For the Prime Numbers game, roll two number cubes labelled 1 to 6. Then multiply the two numbers. When the product is a prime number, Player A gets 10 points. When the product is a composite number, Player B gets 1 point.
   a) Draw a multiplication table to list the outcomes.
   b) What if the number cubes are rolled 60 times. Estimate the number of points each player might get.
   c) Is this a fair game? Explain.
   d) Could you draw a tree diagram to list the outcomes? Explain.

5. Merio’s Deli offers sandwiches on rye or whole wheat bread with one choice of meat: turkey, ham, or pastrami; and one choice of cheese: mozzarella, Swiss, or cheddar. Mya could not make up her mind. She requested a sandwich that would surprise her.
   a) Draw a tree diagram to list the possible sandwiches.
   b) What is the probability Mya will get each sandwich?
      i) Swiss cheese on rye bread
      ii) cheddar cheese but no turkey
   Justify your answers.
6. In a board game, players take turns to spin pointers on these spinners. The numbers the spinners land on are multiplied. The player moves that number of squares on the board.

8. Explain how to simulate the birth of a male or female puppy with each item. Justify your answer.
   a) a coin
   b) a number cube
   c) a spinner

9. According to a news report, 1 out of every 4 new computers is defective.
   a) Design an experiment to estimate the probability that, when 6 new computers are delivered to a store, 3 of them are defective.
   b) Conduct the experiment. What is your estimate of the probability? Justify your answer.

10. Two number cubes are rolled. Each cube is labelled to 1 to 6. The numbers that show are subtracted.
    a) What are the odds in favour of:
       i) the difference is 1?
       ii) the difference is greater than 3?
    b) What are the odds against:
       i) the difference is an odd number?
       ii) the difference is 5?

11. Irina and Reanna share a dirt bike. They toss a coin to decide who rides first each day. Irina picks heads the first day, tails the second day, and tails the third day.
    a) Draw a tree diagram to list the outcomes for 3 coin tosses.
    b) What is the probability Irina will be the first to ride the dirt bike three days in a row?

11. The probability of Anna hitting a home run is 25%. What are the odds against Anna hitting a home run?
1. A coffee shop states that if you buy an extra large coffee, you have a 15% chance of getting a free bagel. Out of 45,550 extra large coffee cups, 7280 cups have the word FREE BAGEL written under the rim.
   a) What is the probability of getting a free bagel if you buy an extra large coffee?
   b) Is the coffee shop's statement correct? Explain.

2. Weng-Wai and Sarojinee make up games that can be played with two tetrahedrons labelled 2 to 5.
   In Weng-Wai's game, players add the numbers rolled.
   If the sum is even, Player A gets 1 point.
   If the sum is odd, Player B gets 1 point.
   In Sarojinee's game, players multiply the numbers rolled.
   If the product is even, Player A gets 1 point.
   If the product is odd, Player B gets 1 point.
   a) What is the probability that Player A will win in Weng-Wai's game? Sarojinee's game?
   b) Is each game fair? Justify your answer.
   c) How could you use tree diagrams to solve this problem?

3. Keyna used this tangram as a dart board. Calculate the probability that a randomly thrown dart will land in each area. Justify your answers.
   a) orange
   b) purple
   c) blue or yellow
   d) green or orange

4. Describe a simulation you could conduct to estimate the probability that 3 people with birthdays in September were all born on an odd-numbered day.
A company that makes fortune cookies has 4 different messages in the cookies. There are equal numbers of each message in a batch of cookies. Design a simulation you could use to solve these problems.

- What if you have 2 fortune cookies. What is the probability you will get 2 different messages?

- What if you have 3 fortune cookies. What is the probability you will get 3 different messages?

- What if you have 4 fortune cookies. What is the probability you will get 4 different messages?

- Estimate how many cookies you will need to get 4 different messages.
Conduct each simulation. Use any materials you think will help. Explain your choice of materials. Answer each problem. Show your work.

**Check List**

Your work should show:
- the steps and procedures you used in your simulations
- an explanation of why you chose each simulation
- all records and calculations, in detail
- clear explanations of your results, with correct use of the language of probability

**Reflect on the Unit**

List the different methods you have learned to estimate and calculate probability. Include an example of how each method is used.
Work with a partner.

Four integer cards, labelled $-3, -2, +1,$ and $+3,$ are placed in a bag.

James draws three cards from the bag, one card at a time. He adds the integers.

James predicts that because the sum of all four integers is negative, it is more likely that the sum of any three cards drawn from the bag will be negative.

In this *Investigation*, you will conduct James' experiment to find out if his prediction is correct.

![Integer cards: -3, -2, +1, +3]

**Part 1**

- Place the integer cards in the bag.
  
  Draw three cards and add the integers.
  
  Is the sum negative or positive?
  
  Record the results in a table.

<table>
<thead>
<tr>
<th>Integer 1</th>
<th>Integer 2</th>
<th>Integer 3</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Return the cards to the bag. Repeat the experiment until you have 20 sets of results.

- Look at the results in your table. Do the data support James' prediction? Explain.
Combine your results with those of 4 other pairs of classmates.
You now have 100 sets of results.
Do the data support James' prediction?
Explain.

Use a diagram or other model to find the theoretical probability of getting a negative sum.
Do the results match your experiment?

Do you think the values of the integers make a difference?
Find 4 integers (2 positive, 2 negative) for which James' prediction is correct.

Part 2
Look at the results of your investigation in Part 1.
➢ If the first card James draws is negative, does it affect the probability of getting a negative sum?
Use the results of Part 1 to support your thinking.

➢ If the first card James draws is positive, does it affect the probability of getting a negative sum?
Use the results of Part 1 to support your thinking.

Take It Further
➢ Emma wonders if she will get a different result if she multiplies the three integers instead of adding them.
She predicts that she is more likely to get a negative product than a positive product.

➢ Design an experiment to find out if Emma's prediction is correct.
Perform the experiment. Was Emma's prediction correct?
Explain.